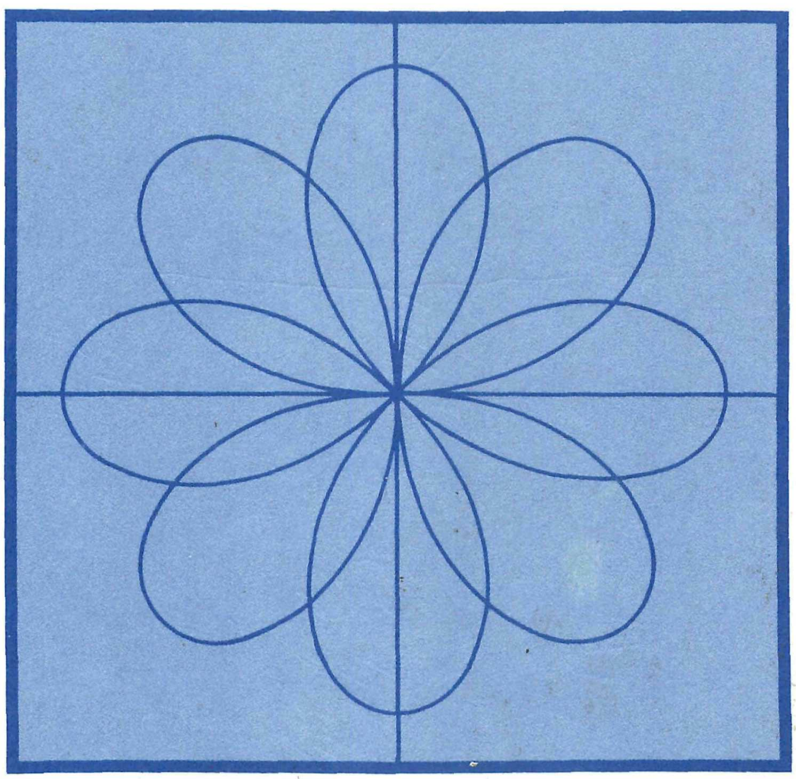
Master Grapher and 3D Grapher

VERSION 1.0



A powerful, interactive graphing utility for functions, polar equations, parametric equations, conic equations, and functions of two variables

Bert K. Waits • Franklin Demana

Master Grapher
and
3D Grapher
Version 1.0

Software Concept and Design
Bert K. Waits, Franklin Demana
The Ohio State University

Apple II Programming
Greg Ferrer
IBM and Macintosh Programming
David B. New

User's Guide
Amy Edwards, David B. New, Chuck Vonder Embse

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3. Information about using this graphing software

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Professor Bert Waits
The Ohio State University
Department of Mathematics
231 W. 18th Avenue
Columbus, Ohio 43210

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David New (*The Ohio State University*)
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The existence of this guide should not be considered as an endorsement of any IBM or Apple product.

Franklin Demana Bert K. Waits

Columbus, Ohio June, 1989

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CHAPTER 1 OVERVIEW

1.1 Introduction

The *Master Grapher* package contains powerful utilities for graphing functions, conics, polar equations, parametric equations, and surfaces (functions of two variables) on Apple II, IBM, and Macintosh computers. This guide contains a chapter for each version - Apple II, IBM, and Macintosh.

1.2 Objectives of Computer Based Graphing

A computer drawn graph is accurate and easy to obtain. Computer drawn graphs can be used as tools to solve equations and enhance the teaching, learning, and understanding of mathematics. Listed below are several important objectives for an interactive computer graphing approach to learning mathematics.

THROUGH THE SPEED AND POWER OF COMPUTERS,
YOU CAN INVESTIGATE MANY EXAMPLES QUICKLY
AND MAKE AND TEST GENERALIZATIONS
BASED ON STRONG GRAPHICAL EVIDENCE

(1) To study the behavior of functions and relations including conic equations, parametric equations, polar equations, and three–dimensional surfaces (functions of two variables).

(2) To deepen understanding and intuition about a wide variety of functions and relations and to provide a foundation for the study of calculus, statistics, and higher mathematics.

(3) To graphically determine the number of solutions to equations and systems of equations. To solve equations, systems of equations, and inequalities graphically with accuracy equal to any numerical approximation method.

(4) To determine relative maximum and minimum values of functions graphically with accuracy equal to any numerical approximation method.

(5) To graphically investigate and determine the solution to "real world" problem situations that are normally not accessible to precalculus students.

(6) To provide geometric representations for problem situations and to analyze their connections with algebraic representations for the problem situations.

COMPUTER GRAPHING IS A FAST AND EFFECTIVE TOOL THAT
YOU CAN USE TO EXPLORE MATHEMATICS AND SOLVE PROBLEMS

1.3. Definitions

The following are definitions are used in this guide.

(1) The **viewing rectangle** [L, R] by [B, T] (see Figure 1.1) is the rectangular portion of the coordinate plane determined by $L \leq x \leq R$ and $B \leq y \leq T$. The $[-10, 10]$ by $[-10, 10]$ viewing rectangle is called the **standard viewing rectangle**. We also use Xmin for L, Xmax for R, Ymin for B, and Ymax for T.

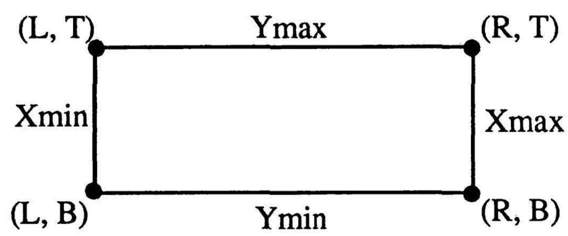

Figure 1.1

(2) **Zoom in** is a process of framing a small rectangular area within a given viewing rectangle, making it the new viewing rectangle, and then quickly replotting the graph in this new viewing rectangle. This feature permits the user to create a decreasing sequence of nested rectangles that "squeeze down" on a key point on a graph. Zoom-in is very useful for solving equations, inequalities, systems of equations and inequalities, and for determining maximum and minimum values of functions.

(2) **Zoom out** is a process of increasing the absolute value of the viewing rectangle parameters. It is important to be able to zoom out in *both* the horizontal and vertical directions at the same time, in *only* the horizontal direction, or in *only* the vertical direction. The zoom-out process is useful for examining limiting, end behavior of relations, and for determining "complete" graphs of relations.

(4) A **complete graph** is either the entire graph or a portion of a graph that shows all of the important behavior and features of the graph. For example, the graph of the relation $x^2 + y^2 = 16$ in $[-10, 10]$ by $[-10, 10]$ is complete because it is the entire graph of $x^2+y^2 = 16$. The graph of $f(x) = x^3-x+15$ in $[-10, 10]$ by $[-10, 30]$ is a complete graph because we can see all of its local extremum values and real zeros. Of course, it is possible to create a function for which you cannot determine *one* viewing rectangle that gives a complete graph. Thus, several viewing rectangles may be needed to describe a complete graph.

(5) The **error** in using a particular point (x, y) in the viewing rectangle [L, R] by [B, T] to approximate any other point (a, b) in the viewing rectangle is *at most* $R - L$ for x and $T - B$ for y. There are also better error bounds possible by using scale marks appearing in a viewing rectangle.

1.4 SYSTEM REQUIREMENTS

Apple II

System requirements:
Apple II+, IIe, IIc.

Programming:
Master Grapher is written in Assembly Language.

IBM

System requirements:
IBM PC or PS/2 or clone. Minimum 256K. *Master Grapher* requires a graphics card (Hercules, vga, cga, ega).

Programming:
Master Grapher is a compiled, stand-alone application.

Macintosh

System requirements:
Macintosh 512, 512E, Plus, SE, or II.

Note: Disk 1 is for 800K machines. Disk 2 is for 512K machines.

Programming:
Master Grapher is a compiled, stand-alone application.

1.5 USER FEEDBACK

The developers invite users of *Master Grapher* to make suggestions they feel would improve future versions. Reporting of any bugs or errors would also be appreciated.

Write:

Bert K. Waits and Franklin Demana
c/o Mathematics Department
The Ohio State University
231 W. 18th Ave.
Columbus, Ohio 43210

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Chapter 2
Apple II Version of *Master Grapher* (version 1.0)

2.1 Starting Up: Remove the *Master Grapher* disk from its envelope and insert it in disk drive #1 with the label up and the oval slot forward (see figure 2.1).

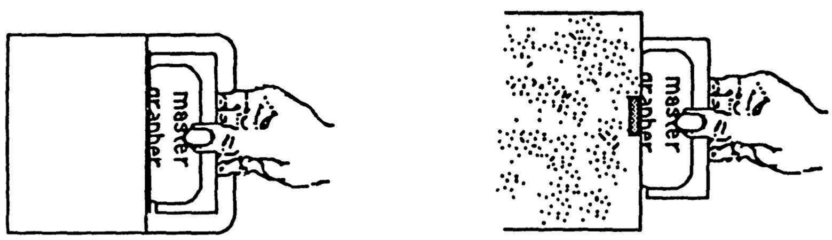


Figure 2.1: Insert *Master Grapher* disk in disk drive #1

Carefully push the disk all the way into the disk drive and then close the gate on the drive until it clicks into place. Turn on the computer and the monitor. *Master Grapher* will automatically run and the title page will be visible (see Figure 2.2).

```
B. Waits or F. Demana
Department of Mathematics
The Ohio State University
231 W. 18th Ave.
Columbus, Ohio
43210

Version 1.0

This disk contains five graphing
programs.  They are all copyrighted,
© 1987, 1988, and 1989 by Bert Waits
and Frank Demana.  The disk or the
programs may not be copied without
written permission from Waits or Demana.
For permission to copy, write to the
above address.

Press any key to go to the main menu.
```

Figure 2.2 : *Master Grapher* title page

Press any key to continue to the Main Menu (see Figure 2.3) to select the graphing program you wish to use.

```
Please choose one:

1) Run the Function Grapher
2) Run the Conic Grapher
3) Run the Parametric Grapher
4) Run the Polar Grapher
5) Run the 3D Surface Grapher

6) Run the Instructional Grapher
7) See instructions

Press the key of your selection (1-7):
```

Figure 2.3: Main Menu showing available graphing programs and instructions

Select the graphing program of your choice by pressing the numbers 1 through 5. Choice # 6 is a self-running demonstration of the **Function Grapher** program. Choice # 7 provides written instructions about the operation of the programs.

2.2 Apple II Function Grapher: Select #1 from the **Main Menu** to run the **Function Grapher** program. After a short pause while the program loads, a title screen will appear (see Figure 2.4). This screen asks the question "Do you have a color monitor (Y/N)?" If you are using a color monitor, respond by keying [Y] and if you are using any other type of monitor respond by keying [N]. Operation of the program will not be affected if you respond with a [Y] and you are not using a color monitor. In some cases, specifying a color monitor when really using a green-screen monitor can help students see slight differences between functions. If you are making printouts of the screen, selecting [N] may give better results.

```
Function Grapher version 1.0

Concept and design by Bert Waits and
  Frank Demana

Machine Language programming by Greg
  Ferrar, Senior at Worthington High
  School

Copyright © 1986, 1987, 1988, 1989 by
  Bert Waits and Frank Demana

Do you have a color monitor (Y/N)?
```

Figure 2.4: Title screen for the Function Grapher

Immediately after entering the information about the type of monitor being used, the **graphing screen** will appear and the graph of the default function $f(x) = sin\ (x)$ will be drawn. Figure 2.5 shows the **graphing screen** with the default **viewing rectangle** of [-10, 10] by [-10, 10]. The values corresponding to the **x-minimum, x-maximum, y-minimum,** and **y-maximum** of the viewing rectangle are displayed on the screen in the corresponding places around the **graphing window.**

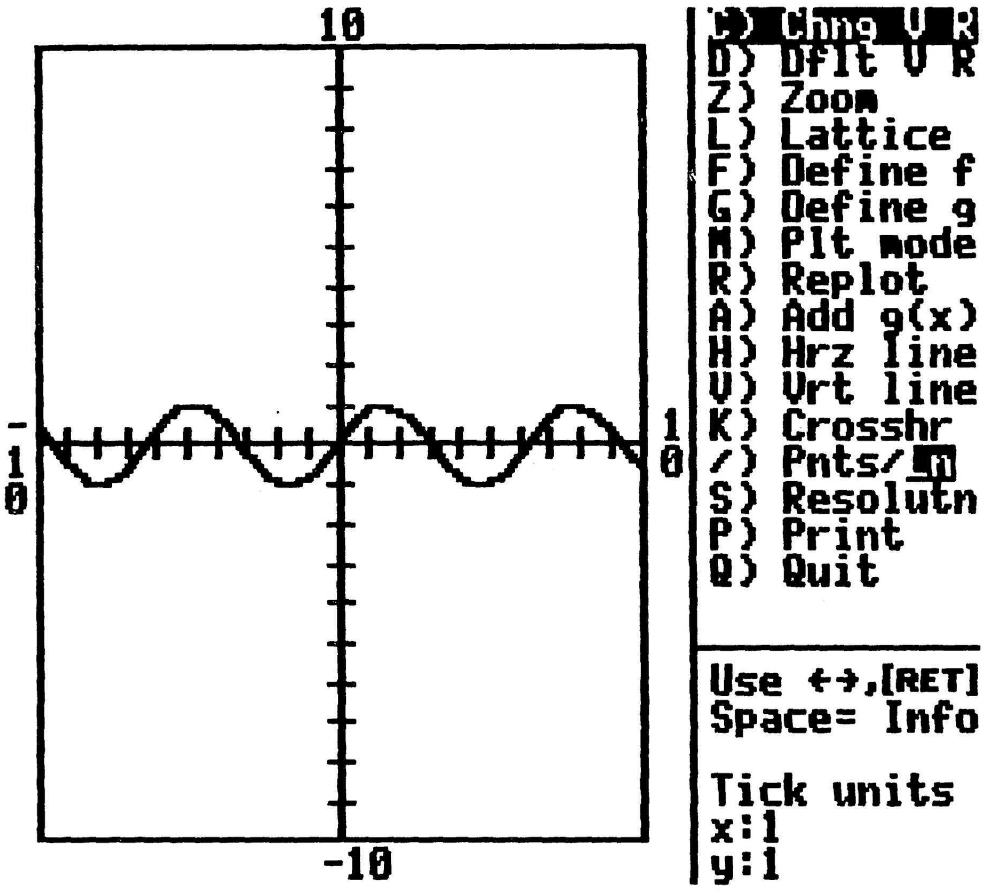


Figure 2.5: Graphing Screen showing the graph and the commands menu

The right side of this screen contains the **commands menu**. Commands are selected by keying the letter or symbol listed for each command, or by using the arrow keys to slide the highlighter bar to a command and pressing [Return]. Below the **commands menu** is a window containing information about selecting commands and displaying information about the functions which are graphed. The information about "**Tick units**" in the lower right corner of the graphing screen indicates the distance between each tick mark on the horizontal (x) and vertical (y) axes. Pressing the [Space Bar] while a graph is plotting will pause the graph and give the coordinates of the last point plotted (see Figure 2.6). Press any key to continue. Pressing the [Space Bar] at any other time will display a window showing the last function or functions which were graphed. Press any key to return to the **commands menu** and erase the **function display window**. Figure 2.7 shows the screen with this function display window activated.

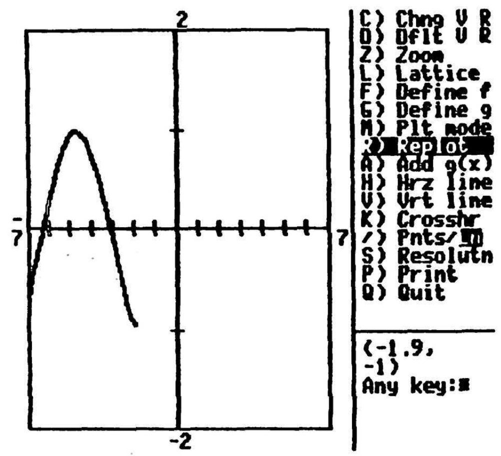


Figure 2.6: Graph paused

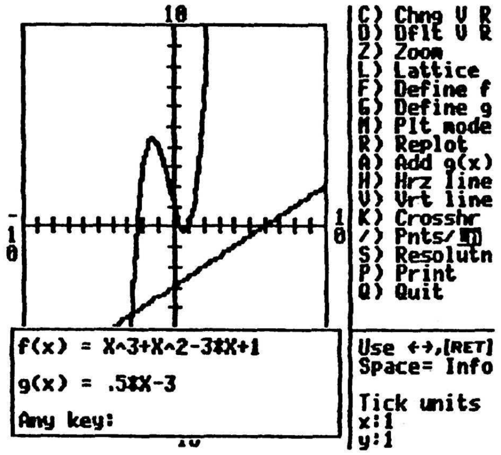


Figure 2.7: Function information window

The definition for the commands which appear in the menu at the right of the graphing screen are as follows:

C) **Chng V R** (Change Viewing Rectangle). This command is used to change the the visible viewing rectangle by entering new values for **x-min, x-max, y-min,** and **y-max.**

D) **Dflt V R** (Default Viewing Rectangle). This command will replot the graph(s) in the default viewing rectangle of [-10, 10] by [-10, 10]. Graphs drawn depend on the **Plot mode** chosen (see **M) Plt mode** below).

Z) **Zoom** This command activates the **Zoom Menu** which controls the zoom-in and zoom-out functions of the program.

L) **Lattice** This command will overlay a lattice (an array of dots) in the viewing rectangle. The dots correspond to the tick marks on the two axes. The distance between the dots in the horizontal or vertical direction corresponds to the values for the "Tick units" found in the lower right corner of the screen.

F) **Define f** Use this command to input a new function, *f*. This function will be plotted immediately in the default viewing rectangle unless **A) Add f(x)** from the **Plot Options** menu is active.

G) **Define g** Use this command to input a second function, *g*. To display the graph of this second function, select **A) Add g(x).**

M) **Plt mode** (Plot mode). This command allows the user to specify which graphs (ie. f, g, f^{-1}, g^{-1}) are automatically drawn on the screen. Once a mode is set, it remains active until changed by the user.

R) **Replot** This command replots the current function(s) in the current viewing rectangle. The current **Plot mode** (see **M) Plt mode** above) determines which graphs will be replotted.

A) **Add g(x)** Select this command to overlay the function *g* on the current screen. The function *g* will be displayed along with any other functions currently displayed. You may add as many *g* functions as you wish by using the **G) Define g** and **A) Add g(x)** commands to overlay graphs. Only the last *g* function remains in the memory of the computer.

H) **Hrz line** (Horizontal line). This command places a moving horizontal line on the graphing screen. Use the [↓] [↑] keys, or the [U] (up) and [D] (down) keys to move this line to any position on the screen. Pressing the [Space Bar] causes the *y*-intercept of the horizontal line to be displayed. Pressing the [G] key allows the user to enter a *y*-coordinate for the horizontal line to "GOTO"; press [Return] to put the horizontal line at the selected position. Press [Esc] to return to the **commands menu.**

V) **Vrt line** (Vertical line). This command places a moving vertical line on the graphing screen. Use the [←] [→] keys, or the [L] (left) and [R] (right) keys to move this line to any position on the screen. Pressing the [Space Bar] causes the *x*-intercept of the vertical line to be displayed. Pressing the [G] key allows the user to enter an *x*-coordinate for the vertical line to "GOTO"; press [Return] to put the vertical line at the selected position. Press [Esc] to return to the **commands menu.**

K) **Crosshr** (Crosshairs). This command combines the moving vertical and horizontal lines so that the coordinates of any point on the screen can be estimated. Use the [←] [→] [↓] [↑] keys or the [L] [R] [U] [D] keys to move the two lines to the desired position. Press the space bar to display the coordinates of the intersection of the two lines (shown in the lower right hand corner of the screen). Press [G] to specify a point for the crosshairs to "GOTO" and press [Return] to send them to that point. Press [Esc] to return to the **commands menu.**

/) **Pnts/Ln** (Points / Lines). This command selects one of two graphing modes, points or line segments. When the Pnts mode is selected, the program will only plot the points (x, y) evaluated for a given function. When the Ln mode is selected, consecutive pairs of points will be connected with a line segment. The Ln mode is the default mode.

S) **Resolution** The resolution of the graph refers to the density of the displayed points. This command allows the user to increase or decrease the density of points in the domain of the function chosen to be evaluated. The result of increasing the density of points is to produce a more accurate graph. However, the time required to draw the denser graph will increase because of the increased number of points which are evaluated.

P) **Print** This command allows the user to print the current graph on paper. To produce a screen-dump of the entire screen including the graph and **commands menu**, press [Control] [P] (at the same time).

Q) **Quit** This command allows the user to exit the program by rebooting the disk. Use this option to return to the **Main Menu** to select another graphing program or to end the session.

2.3 Detailed Guide to Using the Interactive Menu Driven Commands : Use the instructions on "**Starting Up**" in section 2.1 to run *Function Grapher*. The first graphing screen for *Function Grapher* will show the graph of the default function $f(x) = \sin x$ in the default viewing rectangle (see Figure 2.5).

C) **Chng V R** (Change Viewing Rectangle): This command is used to change the portion of the coordinate plane visible in the graphing window. This visible portion of the plane is known as the **viewing rectangle** and is designated by the parameters **x-minimum, x-maximum, y-minimum,** and **y-maximum.** These parameters correspond to the horizontal and vertical dimensions of the viewing rectangle and are displayed in their appropriate positions along the edges of the viewing rectangle.

Suppose we want to view the graph of $f(x) = \sin x$ in the [-7, 7] by [-2, 2] viewing rectangle. To change the dimensions of the viewing rectangle, select the **C) Chng V R** command and the menu at the right of the viewing rectangle will be replaced with a prompt asking for a new value for **x-min;** enter a value for **x-min** and press [Return]. The next prompt will ask for a value for **x-max;** enter a value and press [Return]. Repeat the same procedure for values of **y-min** and **y-max.** After the last value is entered, the graph will be redrawn in the new viewing rectangle. Figure 2.8 shows the screen with the new viewing rectangle of [-7, 7] by [-2, 2] entered. Figure 2.9 shows the graph of $f(x) = \sin x$ redrawn in the new viewing rectangle.

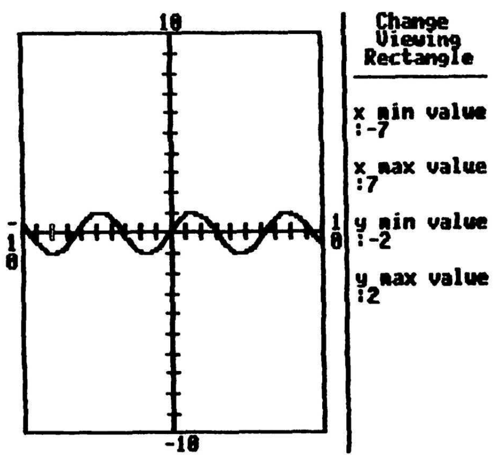


Figure 2.8: Enter new parameters

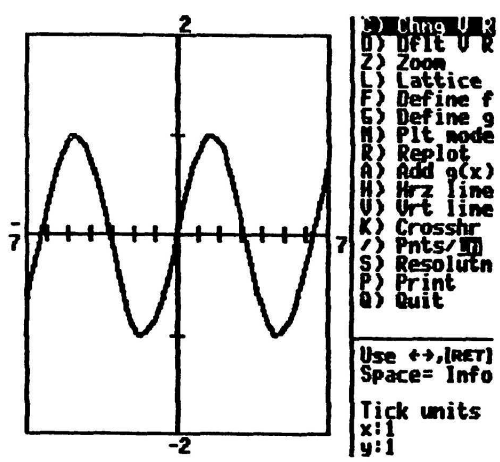


Figure 2.9: Graph redrawn

If you make a mistake when entering a value before pressing Return, press the ← key to backspace and erase characters. Press the Esc key to return to the **commands menu** without changing the current viewing rectangle.

D) Dflt V R (Default Viewing Rectangle): This command will replot the graph(s) in the default viewing rectangle of [-10, 10] by [-10, 10]. Graphs drawn depend on the **Plot mode** chosen (see **M) Plot mode** below).

Z) **Zoom** : Select this option to activate the **Zoom Menu** to zoom in or zoom out on the graph. The **Zoom Menu** appears in place of the **commands menu** at the right of the graph and shows the four zoom options available in the program (see Figure 2.10). To select an option, key the corresponding letter or position the highlight bar and press Return.

I) Zoom in This command allows you to make a *close-up* or *magnified* view of a portion of the graph by capturing that area in a rectangular box or window. Select **I) Zoom in**; a small crosshair (+) will appear in the center of the screen and you will be asked to "Pick a corner of the new window." Move the crosshair to a corner of the new window using the ← → ↓ ↑ keys or the L R U D (left, right, up, down) keys (see Figure 2.10) When the crosshair is in position, press the Space Bar to set this corner. Using the arrow keys, stretch the new viewing window horizontally and vertically to capture the area of the graph you want to examine more closely (see Figure 2.11). When you are satisfied with the location of the viewing window, press the Space Bar and the function will be replotted in the new viewing rectangle (see Figure 2.12). The values for **x-min, x-max, y-min,** and **y-max** around the edges of the screen now represent the new viewing rectangle established by the rectangular window. Press Esc to cancel the zoom process and return to the **commands menu.**

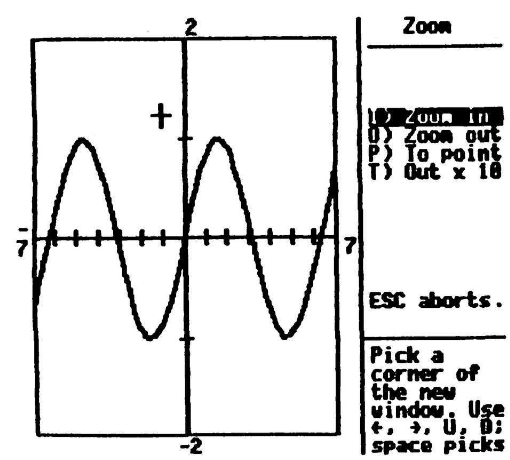


Figure 2.10: Select a corner

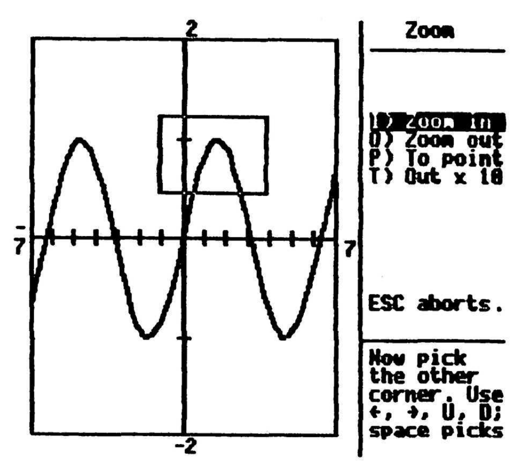


Figure 2.11: Select the diagonal corner

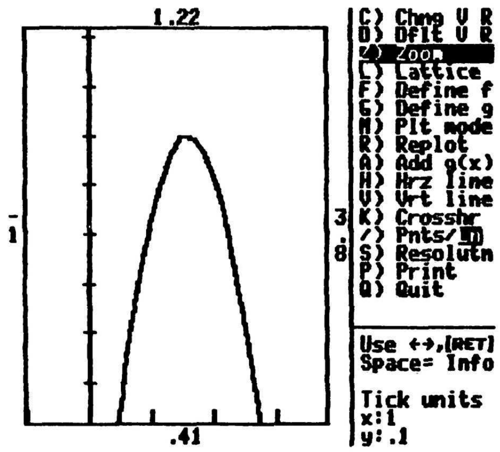


Figure 2.12: Graph redrawn in the new viewing rectangle

O) Zoom out: This command allows you to zoom-out, or back away from a graph to see a larger portion of the coordinate plane. The zoom-out is done relative to the center of the screen by the horizontal and vertical factors you specify. Select **O) Zoom out** and the **Zoom Menu** will be replaced with a prompt asking "Please enter the zoom factor for the *x*- axis." Entering the value "5" will multiply the horizontal distance across the screen by 5. Enter a value and press [Return]. Next, the sam prompt for the *y*-axis will appear; enter a value and press [Return]. Figure 2.13 shows the zoom-out prompts with a factor of 5 entered for the *x*-axis and a factor of 2 entered for the *y*-axis. After the two factors are entered, the graph will automatically be redrawn in the new viewing rectangle. Figure 2.13 shows the graph of $f(x) = \sin x$ drawn in the viewing rectangle [-7, 7] by [-2, 2]. Figure 2.14 shows the graph redrawn in the new viewing rectangle [-35, 35] by [-4, 4] after zooming out by a factor of 5 on the *x*-axis and 2 on the *y*-axis.

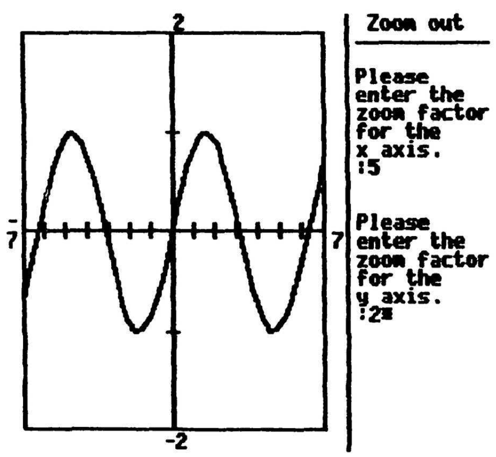


Figure 2.13: Enter Zoom-out factors

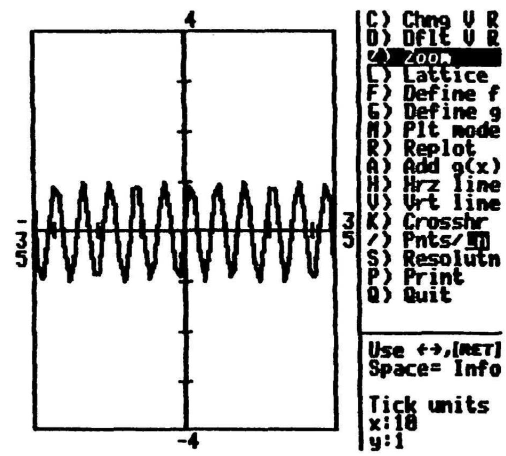


Figure 2.14: Graph redrawn

When you are not sure that the current graph is a complete graph of a function, you can use the **O) Zoom out** command to quickly look for an appropriate viewing rectangle that displays a complete graph, or to determine that the original view was a complete graph. Often you will need to use different zoom factors for the x-axis and the y-axis as we did above.

Note: the **O) Zoom out** command can be used to zoom in by setting the zoom factors between 0 and 1.

P) To point This command allows you to zoom in on a selected point of a graph by a factor of 0.1. Select **P) To point** and a crosshair (+) will appear at the center of the screen. Use the ← → ↓ ↑ keys to move the crosshair to the point you wish to be the center of the new viewing rectangle and press Space Bar. The graph will be redrawn using the selected point as the center of the new viewing rectangle and the distance across the new viewing rectangle will be 1/10 of the previous viewing rectangle (zoom-in by a factor of 0.1). Figure 2.15 shows the crosshair in position on the graph of $f(x) = \sin x$ in the viewing rectangle [-7, 7] by [-2, 2]. This viewing rectangle represents a distance of 14 units in the horizontal and 4 units in the vertical direction. Figure 2.16 shows the graph redrawn in the new viewing rectangle [.91, 2.31] by [.81, 1.21] centered on the selected point. This viewing rectangle represents a distance of 1.4 units in the horizontal and .4 units in the vertical direction.

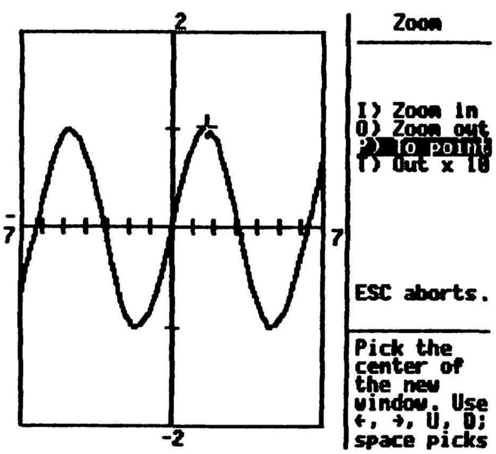


Figure 2.15: Center point selected

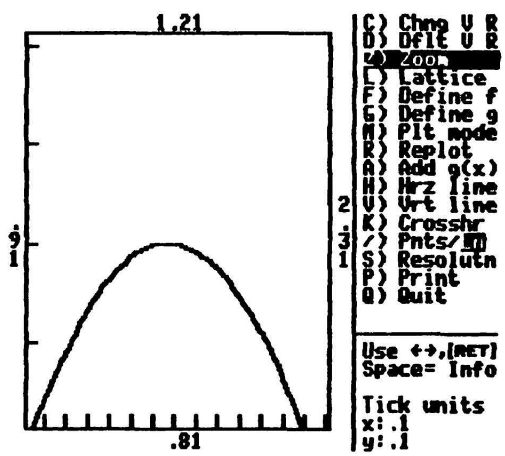


Figure 2.16: Zoom in by factor of 1/10

T) Out x 10 This command zooms out by a factor of 10 relative to the center of the screen. The distance across the screen in the horizontal and vertical direction is multiplied by 10. The center of the new viewing rectangle is the same point as the center of the previous viewing rectangle.

L) Lattice: This command overlays a lattice (an array of dots) in the viewing rectangle. The distance between these dots is given by the **Tick units** , or scale values, found in the lower right-hand corner of the screen. This command is helpful when you want to estimate the coordinates of a point that is not on an axis. For example, select **D) Dflt V R** to draw the graph of $f(x) = \sin x$ in the default viewing rectangle and then zoom in on some portion of the graph using the **Zoom-in** commands. Next, select

L) Lattice to obtain a lattice in the viewing rectangle (see Figure 2.17). The **Tick units** specify the distance between any two dots vertically and horizontally. Now, it is easy to estimate the coordinates of some point on the graph.

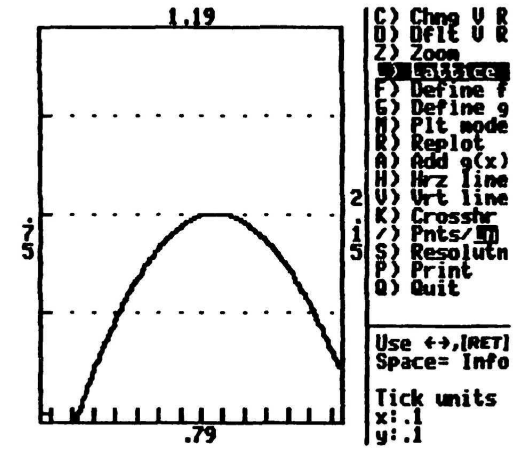


Figure 2.17: Lattice added to the viewing rectangle; Tick unit .1 by .1

F) Define f : This command allows you to enter a new function to be graphed. To enter the new function $f(x) = x^3 + x^2 - 3x + 1$, proceed as follows: select **F) Define f** and a window will appear showing the current f function and a prompt asking you to enter the new f function (see Figure 2.18).

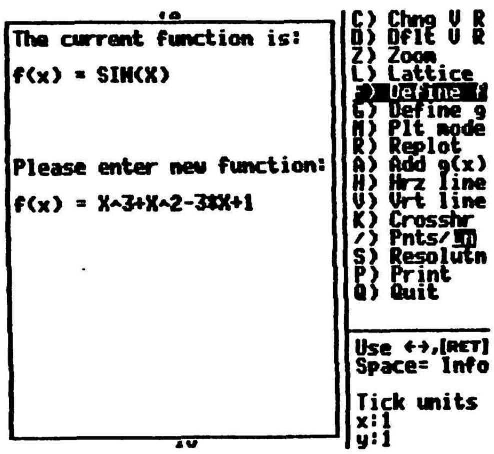


Figure 2.18: Enter the new *f* function

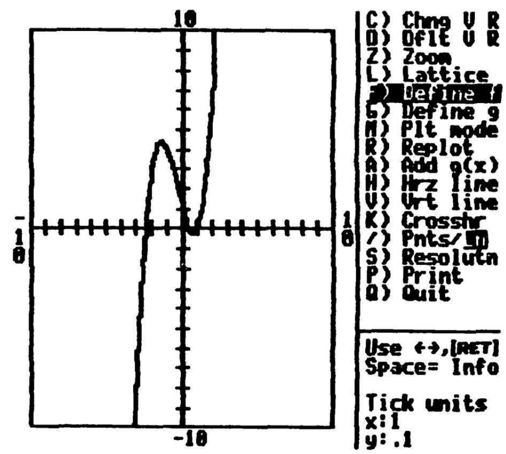


Figure 2.19: New function graphed

We need to enter 3 and 2 as exponents. The Apple computer uses the usual BASIC language notations caret symbol ^ ([Shift] [6]) to indicate exponentiation and the symbol * ([Shift] [8]) to indicate multiplication. (If you are using an Apple II +, use [Shift] [N] for ^ and [Shift] [:] for * .) Type in the keystrokes [X] [^] [3] [+] [X] [^] [2] [-] [3] [*] [X] [+] [1] and press [Return]. The graph of

$f(x) = x^3 + x^2 - 3x + 1$ will be drawn in the default viewing rectangle (unless the **A) Add f(x)** option is selected from the **Plot Options** menu). See Figure 2.19.

Special keying instructions are needed to enter built-in functions or to obtain correct graphs of some special functions. Here are some examples.

1) The absolute value function **| X |** is entered as **ABS (X).**
2) The natural logarithm function ***ln x*** is entered as **LOG (X).** Note: $y = \log_b (x)$ can be graphed for any base b by entering **LOG (X) / LOG (B).**
3) The exponential function e^x is entered as **EXP (X).**
4) The root function $\sqrt[n]{x}$ for ***n*** **odd** is entered as **ABS (X) / X * ABS (X) ^ (1 / N).** You can enter $x^{1/n}$ as *x* ^ (1 / *N*), but for *n* odd, you will only get the portion of the graph in the first quadrant.
5) The greatest integer function $[x]$ is entered as **INT (X).**
6) The signum function is entered as **SGN (X).**
7) The square root function is entered as **SQR (X).**

G) Define g: This command, together with the **A) Add g(x)** command, is used to overlay the graph(s) of additional function(s) in the same viewing rectangle with the current graph of the function *f*. Operation of this command is identical to the **F) Define f** command except that once a *g* function is entered into the computer, its graph will not be drawn automatically; the **A) Add g(x)** command must be used to draw the graph.

If you use the **G) Define g** and **A) Add g(x)** commands several times, then the graph of *f* together with the graphs of all the *g* functions will appear in the same viewing rectangle. Thus, you can overlay the graphs of as many functions as you wish. The program, however, only holds two function definitions at a time, the last *f* and *g* function defined; once a new *f* or *g* function is defined, it replaces any other function entered. If you replot the functions or change the scale of the viewing rectangle, the only functions available are the latest *f* and *g* functions entered. Figure 2.20 shows the screen for entering the *g* function and Figure 2.21 shows the graph of $y = .5x - 3$ overlaid in the viewing rectangle using the **A) Add g(x)** command. Note: The option **A) Add f(x)** from the **Plot Options** menu can do much the same thing.

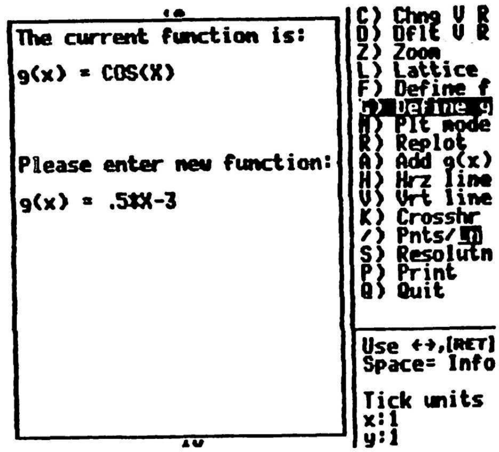


Figure 2.20: Enter new g function

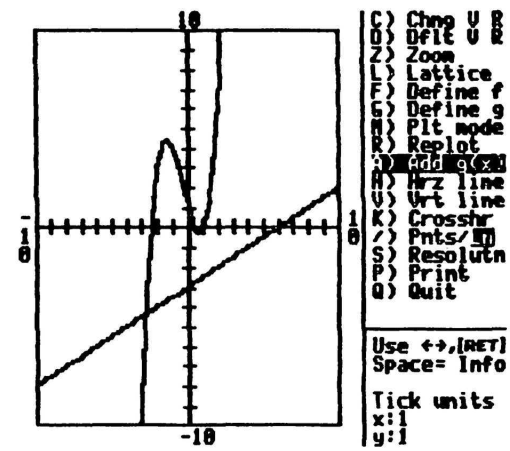


Figure 2.21: Graph of g overlaid

M) Plot mode: Selecting this option activates the **Plot Mode Options Menu** (see Figure 2.22). This menu allows you to specify which functions you would like to plot automatically when graphs are redrawn. For example, selecting option **5) f and g**, causes both currently defined functions, f and g, to be drawn when zooming in or out, or when the **R) Replot** command is selected. (Figure 2.23 shows the effect of using the **R) Replot** command after selecting **5) f and g**.) By selecting **G)** g(x) you could make the g function the default function in all plotting formats. This menu also allows plotting of the inverse relationships $f^{-1}(x)$ and $g^{-1}(x)$ in various combinations. **F)** f(x) is the default setting for the **Plot Mode**. If you select the **G)** g(x) option, then you can overlay the graph of $f(x)$ by selecting the **A) Add** f(x) option from the **Plot Options** menu and then **R) Replot** from the **commands menu**. Once a new **Plot Mode** is selected it stays active until you change it again.

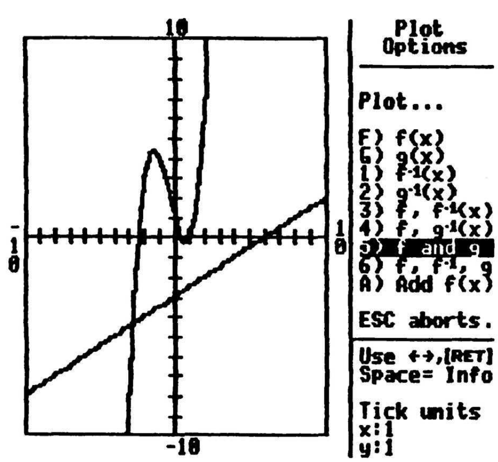


Figure 2.22: Plot Mode Options Menu

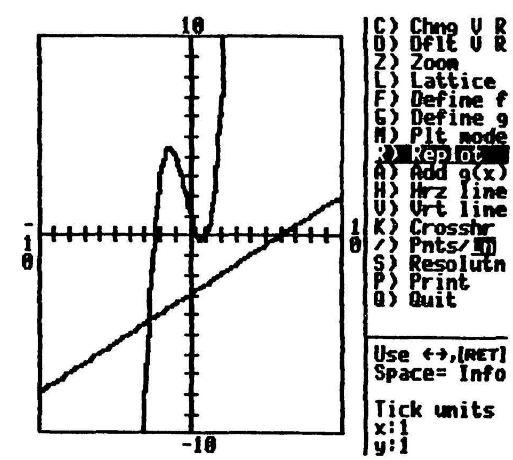


Figure 2.23: Replot both f and g

R) Replot: This command replots the defined function(s) in the current viewing rectangle. The function(s) automatically replotted are those designated by the current **Plot Mode.**

A) Add g(x): This command is used to overlay a function g in the current viewing rectangle. See the command **G) Define g** above.

H) Hrz line: This command is used to draw a moving horizontal line in the current viewing rectangle. The lower right hand corner of the screen contains directions on how to manipulate the line. Use the [↓] [↑] keys (or [U] (up) and [D] (down) keys) to move the line to the desired location (see Figure 2.24). (Note: the horizontal line first appears in the center of the screen; if the x-axis is in the center of the screen, the horizontal line will cover the axis and it will appear that the axis has disappeared. The axis will reappear when the line is moved.) Press the [Space Bar] to get the y-intercept of the line (see Figure 2.25). Press [G] for "GOTO" to send the line to a specific location in the viewing rectangle. Press [Esc] to return to the **commands menu.**

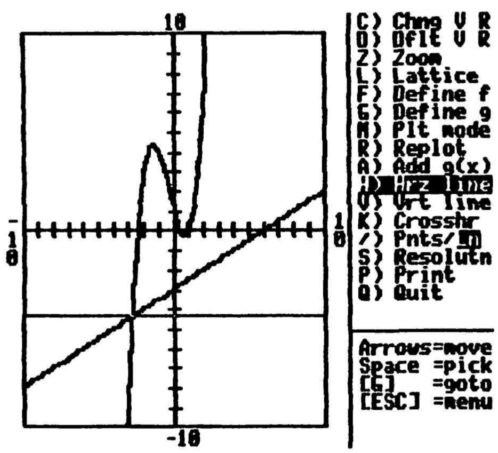


Figure 2.24: Horizontal line in position

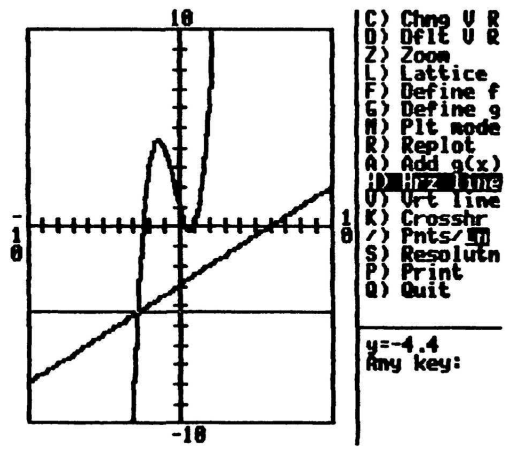


Figure 2.25: Y-intercept is - 4.4

V) Vrt line: This command operates like the **H) Hrz line** except that a moving vertical line is drawn in the viewing rectangle. Use the [←] [→] keys (or [R] (right) and [L] (left) keys) to move the vertical line into position. When the [Space Bar] is keyed, the x-intercept of the vertical line is given in the lower right-hand corner of the screen. Press [Esc] to return to the **commands menu.**

K) Crosshr : This command draws both a moving horizontal and a moving vertical line in the viewing rectangle. Operation of this command is like the **H) Hrz line** and **V) Vrt line.** commands. This command can be used to estimate the coordinates of any point in the viewing rectangle by placing the intersection of the horizontal and vertical lines on that point and pressing the [Space Bar] . Figure 2.26 shows the crosshair positioned at a local maximum value of the graph of $f(x) = x^3 + x^2 - 3x + 1$.

The estimate of that point is (-1.5, 4.4) as seen in the lower right-hand corner of the screen. Press [G] to GOTO a specific point in the viewing rectangle. The program will ask for x and y values to locate the crosshair. Press [Esc] to return to the **commands menu**.

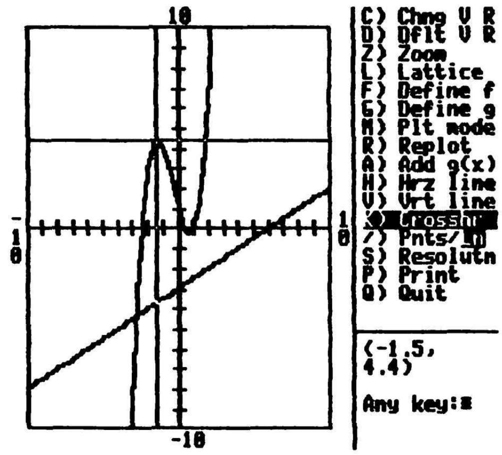


Figure 2.26: Crosshair on a local maximum

/) **Pnts/Ln** : This command gives you the option of drawing a graph by only plotting individual points or by connecting the points with straight line segments. The default setting is /) Pnts/Ln, meaning that graphs will be drawn using line segments to connect each pair of points. The highlighted Ln means that line segments will be used to connect the points. Pnts means that the graph will be drawn as a set of points. Figures 2.27 and 2.28 show the same two graphs drawn with line segments and as points only. The [/] key acts as a toggle switch; each time you press [/] the program switches back and forth between /) Pnts/Lns and /) Pnts/Lns.

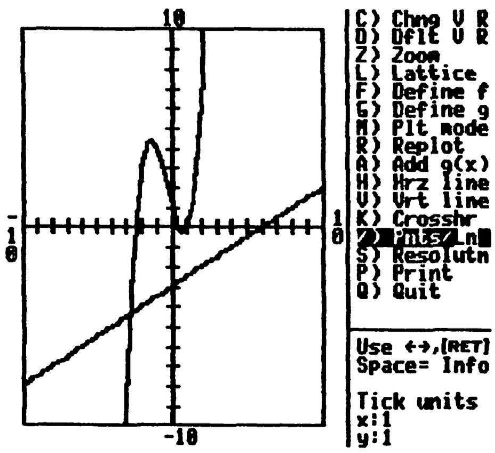


Figure 2.27: Graph drawn with line segments

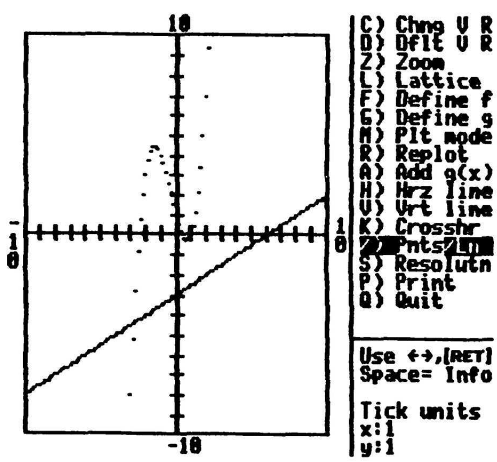


Figure 2.28: Graph drawn as points

S) Resolutn : This command allows you to set the resolution, or density of points to be plotted on your graphs. Larger values for resolution mean more points plotted; smaller values mean less points plotted. In fact, the value specified in the **S) Resolutn** option is the number of points plotted. The speed of the plot is indirectly related to the resolution. As the number of points increases, the speed of the plot decreases, and vice versa. The default resolution of 60 gives fairly accurate graphs, but often it is necessary to increase the resolution to obtain more accurate graphs. Figure 2.29 shows the resolution screen with the prompt asking for an input. The value 120 has been entered. The results of changing the resolution will not be seen until the graph is replotted.

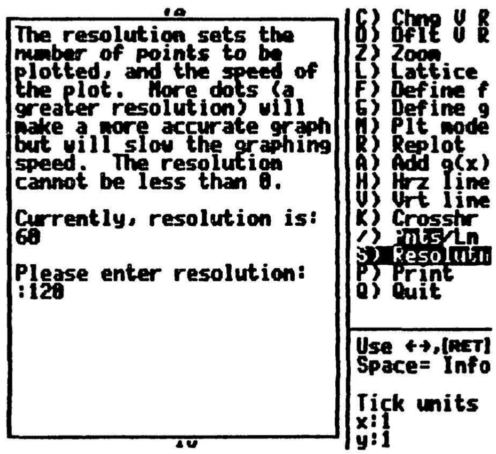


Figure 2.29: Resolution screen with new input of 120

P) Print : This command allows you to obtain a print-out of the current graph. Select the **P) Print** command and the **Print Menu** will appear. Select the type of printer interface card you have in your computer and press [Return]. The program will prompt you to prepare your printer; press any key to print. When the printing is complete, the program will return to the **commands menu**. Press [Esc] to return to the **commands menu** without printing.

To produce a screen-dump of the entire screen including the graph and the **commands menu,** press the [Control][P] keys (at the same time). Follow the prompt at the bottom of the screen in the same way listed above.

Q) Quit : Select this command when you are ready to quit the program or to return to the **Main Menu** to select another graphing program. After you select **Q) Quit**, press any key to go to the **Main Menu**. Press [Esc] to abort the **Q) Quit** command and return to the **commands menu**.

2.4 Apple II Examples

2.4.1 Entering Functions : Try graphing the following functions. A graph and a viewing rectangle are given for each function. If you fail to obtain the same graph, check the keying sequence you used and compare it with the ones given below for the four graphs.

(1) $f(x) = 2x + 1$

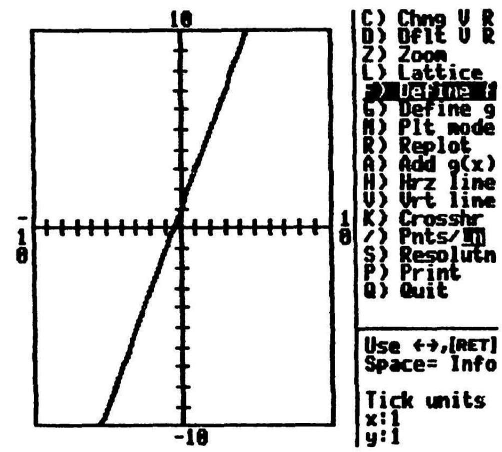


Figure 2.30

(2) $f(x) = x^2 - 3$

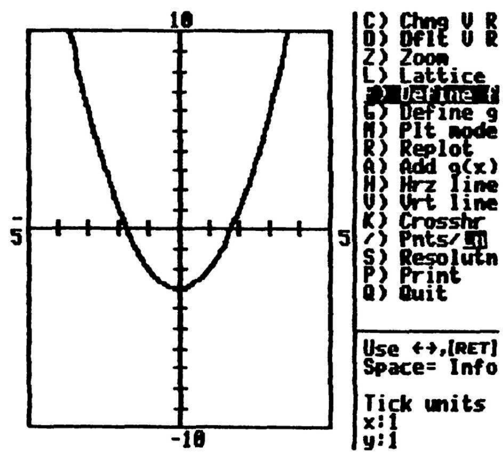
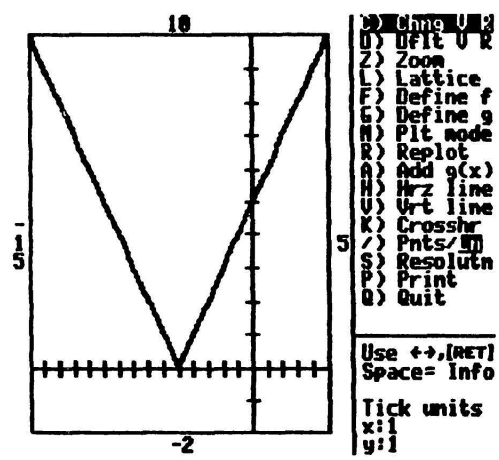


Figure 2.31

(3) $f(x) = |x + 5|$



(4) $f(x) = 2(x^2 + 1)$

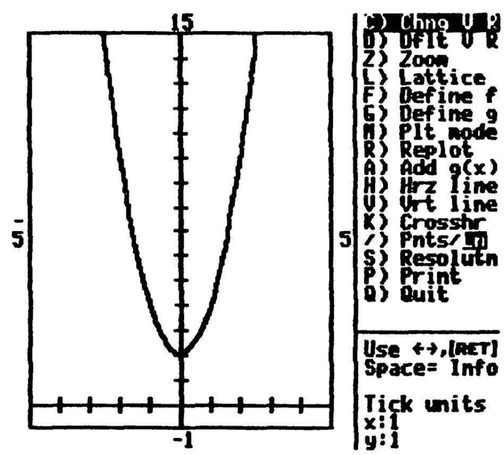


Figure 2.32

Figure 2.33

Were you able to produce these graphs? If not, here are the correct keying sequences for 1 - 4.

(1) $\boxed{2}\boxed{*}\boxed{X}\boxed{+}\boxed{1}$

(2) $\boxed{X}\boxed{\wedge}\boxed{2}\boxed{-}\boxed{3}$

(3) $\boxed{A}\boxed{B}\boxed{S}\boxed{(}\boxed{X}\boxed{+}\boxed{5}\boxed{)}$

(4) $\boxed{2}\boxed{*}\boxed{(}\boxed{X}\boxed{\wedge}\boxed{2}\boxed{+}\boxed{1}\boxed{)}$

(5) $f(x) = 3(x+2)^2$

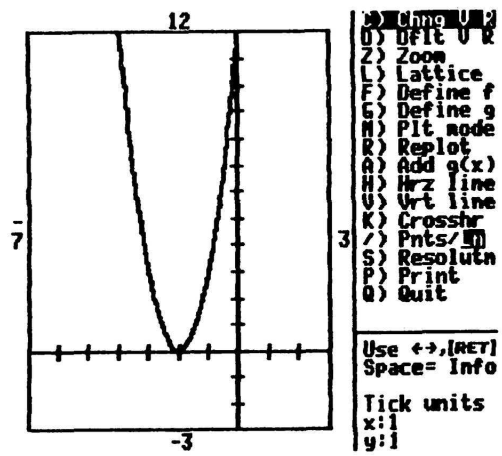


Figure 2.34

(6) $f(x) = (-1)(x-1)^2$

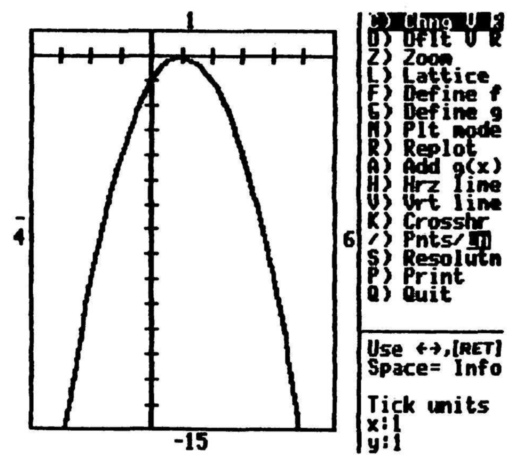


Figure 2.35

(7) $f(x) = \frac{2}{x} + \frac{3}{(x-2)^2}$

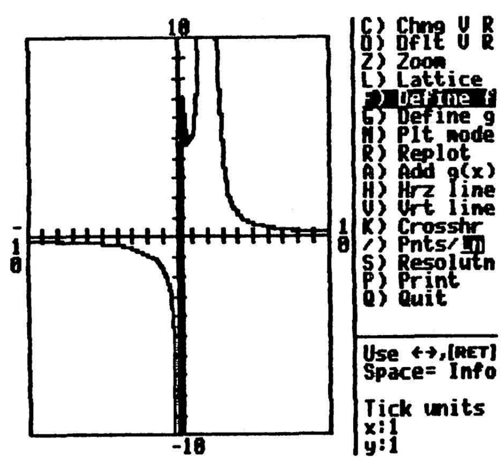


Figure 2.36

(8) $f(x) = \frac{3}{x+1}$

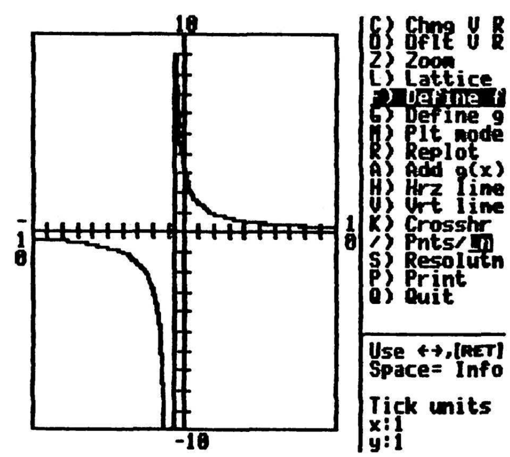


Figure 2.37

Here are the keying sequences needed to draw graphs 5 - 8:

(5) $\boxed{3}\boxed{*}\boxed{(}\boxed{x}\boxed{+}\boxed{2}\boxed{)}\boxed{\wedge}\boxed{2}$

(6) $\boxed{(}\boxed{-}\boxed{1}\boxed{)}\boxed{*}\boxed{(}\boxed{x}\boxed{-}\boxed{1}\boxed{)}\boxed{\wedge}\boxed{2}$

(7) $\boxed{2}\boxed{/}\boxed{x}\boxed{+}\boxed{3}\boxed{/}\boxed{(}\boxed{x}\boxed{-}\boxed{2}\boxed{)}\boxed{\wedge}\boxed{2}$

(8) $\boxed{3}\boxed{/}\boxed{(}\boxed{x}\boxed{+}\boxed{1}\boxed{)}$

2.4.2 Zoom-out : Zoom-out is another way to change the viewing rectangle. Let's look at an example. Change the function f to $f(x) = x^2 - 23x + 132$. The graph of f in the default viewing rectangle of [-10, 10] by [-10, 10] is given in Figure 2.38. Notice that this figure shows very little of the graph of f.

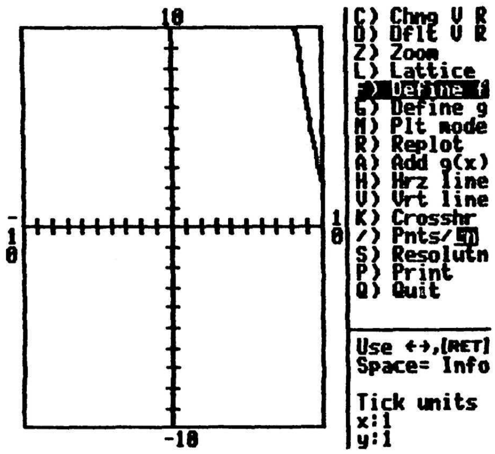


Figure 2.38

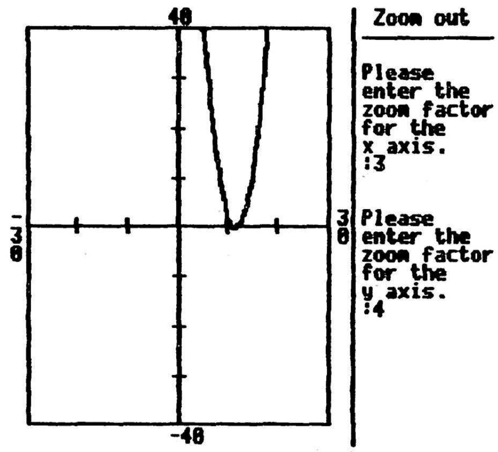


Figure 2.39

Select **O) Zoom out** from the **Zoom menu** and zoom out horizontally (along the x-axis) by a factor of 3, and vertically (along the y-axis) by a factor of 4 to look at the graph in a larger viewing rectangle (see Figure 2.39). Try this on your own first. If you need some help, see the keying sequence below.

1) Select **Z) Zoom** from the **commands menu**
2) Select **O) Zoom out** from the **Zoom menu**
3) Answer the first prompt ("Please enter the zoom factor for the x axis:") with the keys

   [3] [Return].

4) Answer the second prompt ("Please enter the zoom factor for the y axis:") with the keys

   [4] [Return].

When you are not sure you have a complete graph of a function, you can use the **O) Zoom out** command to quickly look for an appropriate viewing rectangle that displays a complete graph of the function. You often need to use a vertical zoom factor different from the horizontal zoom factor. For example, change the function f to $f(x) = -5x^3 + 7x^2 + 6x + 25$. Zoom out using 10 for the zoom factor for both the horizontal and vertical direction. The new viewing rectangle will be [-100, 100] by [-100, 100]. What happened? Select **D) Dflt V R** to return to the default viewing rectangle. Zoom out again using a zoom factor of 1 for the x-axis (horizontal) and a factor of 100 for the y-axis (vertical).

2.4.3 Zoom-in : Change f to the function $f(x) = -x^3 - 4x^2 + 3x + 5$. The graph of f in the default viewing rectangle is shown in Figure 2.40.

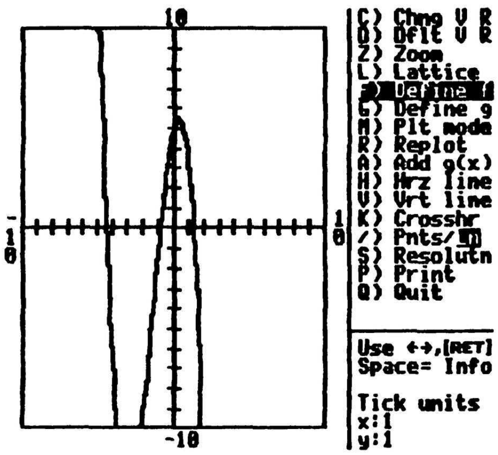


Figure 2.40

In this viewing rectangle the middle x-intercept appears to be very close to -1. Suppose we want a better approximation to this x-intercept. We can use the **I) Zoom in** command to obtain a closer view. We will use this command to form a box around this middle x-intercept; this box will be the new viewing rectangle. Select **I) Zoom in** from the **Zoom menu.** Using the [←] [→] [↓] [↑] keys, or the [L] [R] [U] [D] keys, move the crosshair to one corner of a box which will capture the selected x-intercept (see Figure 2.41). When the first corner is in place press [Space Bar] to set the corner. Next, use the [←] [→] [↓] [↑] keys or the [L] [R] [U] [D] keys to stretch the box in such a way that it will capture the x-intercept under investigation (see Figure 2.42). When the box is set correctly, press [Space Bar] to draw the graph of the function in the new viewing rectangle.

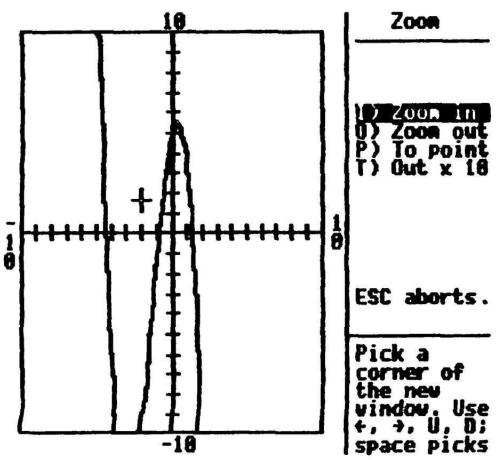


Figure 2.41

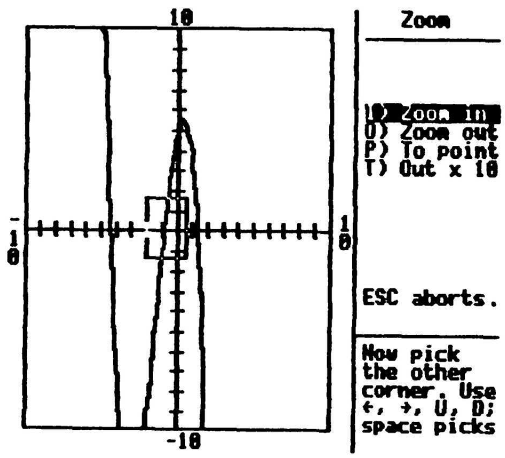


Figure 2.42

Figure 2.43 shows the graph of $f(x) = -x^3 - 4x^2 + 3x + 5$ drawn in the new viewing rectangle. Notice from this magnified view that the x-intercept we were investigating is actually in the interval (-1, 0), and not at -1 as we suspected. You can continue to zoom in until you have located this x-intercept with as much accuracy as desired.

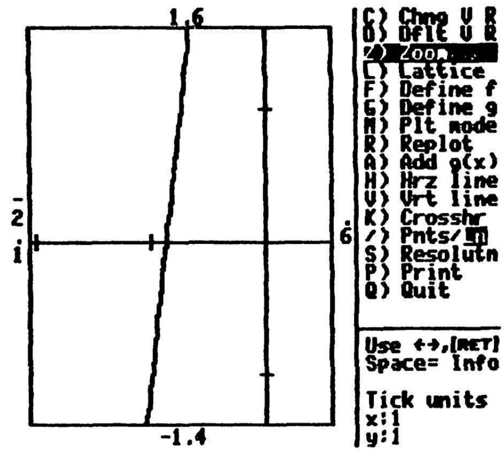


Figure 2.43

2.4.4 Lattice : This command overlays a lattice (an array of dots) in the viewing rectangle. The distance between these dots is given by the "Tick units" found in the lower right-hand corner of the screen. For example, the graph of the function $f(x) = x^2 - 3x + 2$ in the default viewing rectangle appears to flatten out on the x-axis between $x = 1$ and $x = 2$ (see Figure 2.44). Change the viewing rectangle to [1, 2] by [-.5, 0] (see Figure 2.45).

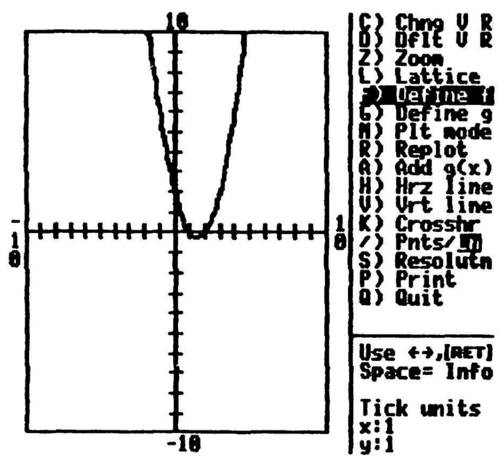


Figure 2.44

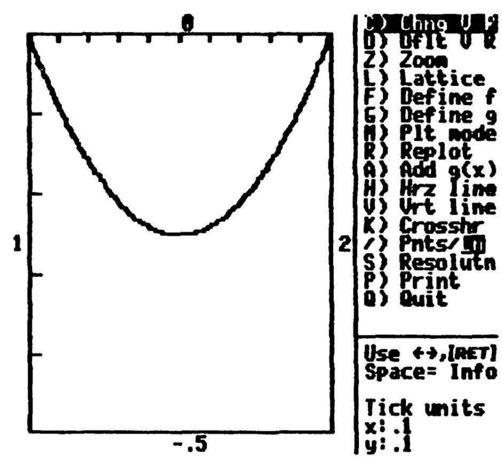


Figure 2.45

We can estimate the coordinates of the vertex of this parabola from the graph in Figure 2.45 but an even better approximation can be obtained after we overlay a lattice by selecting **L) Lattice** from the **commands menu** (see Figure 2.46). The distance between the dots both horizontally and vertically in this figure is 0.1 units. We can now estimate the coordinates of the vertex to be (1.5, -0.25). If you want to remove the lattice, select **R) Replot** to replot the graph.

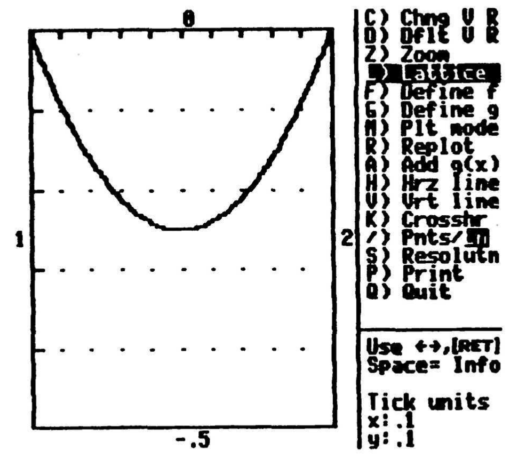


Figure 2.46

2.4.5 Overlaying Additional Functions: Select **F) Define f** and enter $f(x) = x^2$ as the function f. Change the viewing rectangle using the **C) Chng V R** command to [-4, 4] by [-1, 15]. Select **G) Define g** and enter $g(x) = 2x^2$ as the function g. Next, select **A) Add g(x)** to overlay the graph of the function $g(x) = 2x^2$ on the graph of $f(x) = x^2$ (see Figure 2.47).

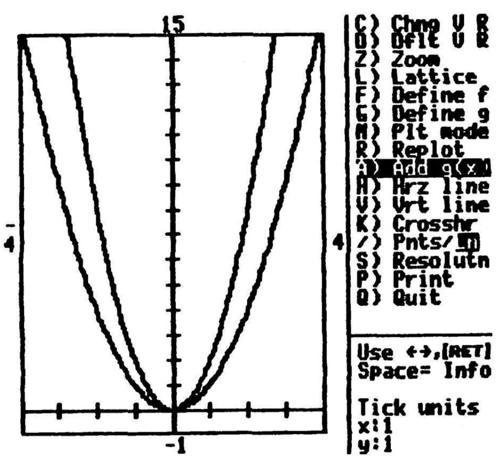


Figure 2.47

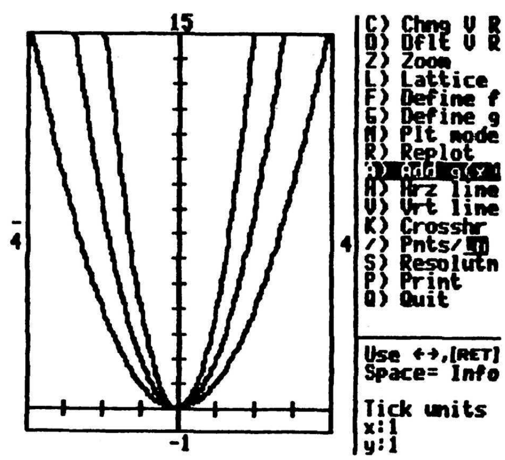


Figure 2.48

Next, overlay the graph of $g(x) = 4x^2$ to the viewing rectangle displaying the graphs of

$f(x) = x^2$ and $g(x) = 2x^2$ (see Figure 2.48). This is done by using the **G) Define g** and **A) Add g(x)** commands again. What happens to the graph of $f(x) = ax^2$ as the coefficient, a, of x^2 gets larger?

Select **R) Replot** to draw only the graph of $f(x) = x^2$ in the present viewing rectangle. Change the viewing rectangle to [-10, 10] by [-1, 12] (see Figure 2.49). Using the **G) Define g** and **A) Add g(x)** commands, add the following function to the viewing rectangle: $g(x) = \frac{1}{2}x^2$, $g(x) = \frac{1}{3}x^2$, and $g(x) = \frac{1}{10}x^2$. The keying sequence to enter the first function is $\boxed{(}\boxed{1}\boxed{/}\boxed{2}\boxed{)}\boxed{*}\boxed{x}\boxed{\wedge}\boxed{2}$. The others are similar.

What can you say about the graph of $f(x) = ax^2$ as the coefficient of x^2 takes on fractional values between 0 and 1? Figure 2.50 shows these four functions overlaid on the same viewing rectangle.

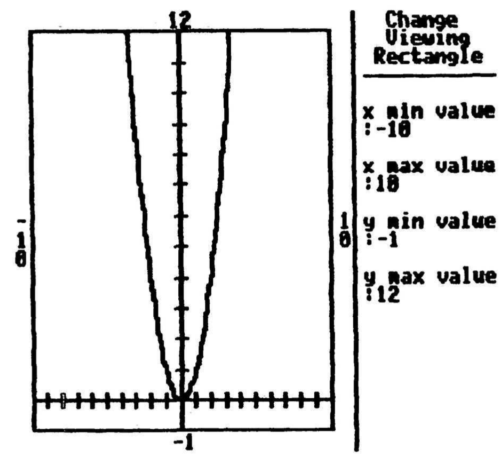


Figure 2.49

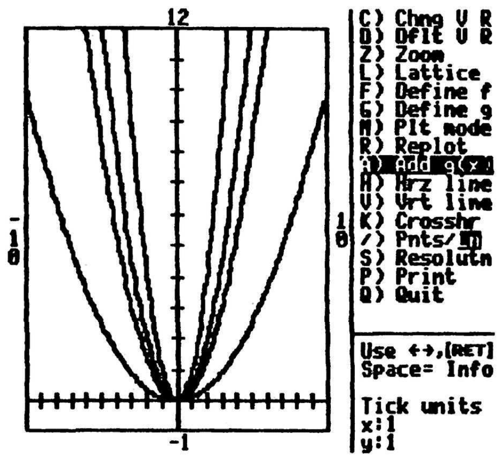


Figure 2.50

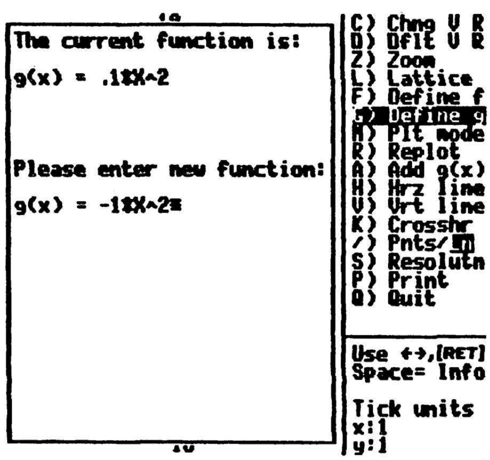


Figure 2.51

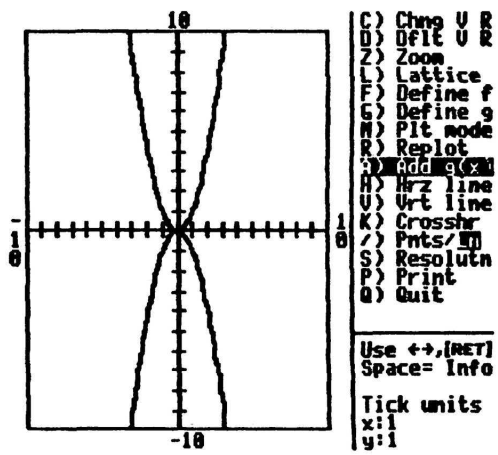


Figure 2.52

Select **D) Dflt V R** to draw the graph of $f(x) = x^2$ in the default viewing rectangle. Overlay the graph of $g(x) = -x^2$ using the the keying sequence [-] [1] [*] [x] [^] [2] (see Figure 2.51). Notice we had to insert a [1] in order to get the correct graph of $g(x) = -x^2$. What we entered then was $g(x) = -1x^2$. Your screen should look like the one in Figure 2.52. What can you say about the graph of $f(x) = ax^2$ if a is negative?

2.4.6 Crosshair Lines : Graph the function $f(x) = x^2 - 3x - 2$ in the default viewing rectangle. Select **K) Crosshr** from the **commands menu.** Use the [←] [→] [↓] [↑] keys or the [L] [R] [U] [D] keys to move the horizontal and vertical crosshair lines so that they intersect at some point of interest in the viewing rectangle. For example, use the crosshair to estimate the coordinates of the vertex of the parabola. Once the crosshairs are in position, press [Space Bar] to see the coordinates of the point of interest. Figure 2.53 shows the crosshairs on the vertex of the parabola and the estimate of (1.4, -4.3) for that point (in the lower right-hand corner of the screen).

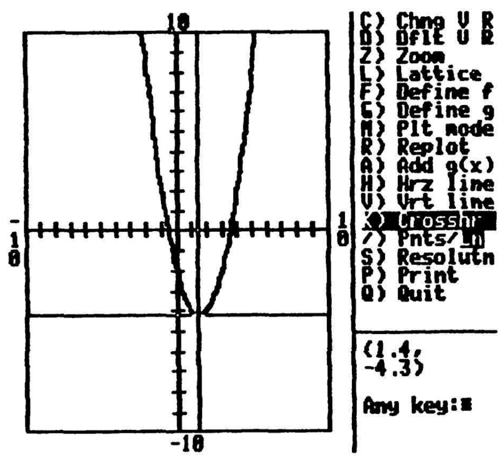


Figure 2.53

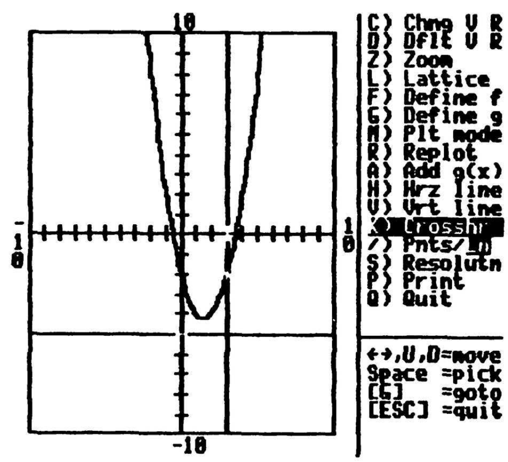


Figure 2.54

To continue, press any key and you can move the crosshairs to another point in the viewing rectangle. To send the crosshairs to a specific location press [G] . A prompt in the lower right-hand corner of the screen will ask "Please type x value to jump to:". Enter an x value contained in the viewing rectangle and press [Return] . A second prompt asking "Please type y value to jump to:" will appear. Enter a y value contained in the viewing rectangle and press [Return]. Figure 2.54 shows the result of entering $x = 3$ and $y = -5$ for the two prompts; the crosshairs intersect at the point (3, -5). You may continue this process as long as you wish. When you have finished using this command, press [Esc] to return to the **commands menu.**

2.4.7 Line and Point Plots : Figure 2.55 shows the graph of $f(x) = \frac{3}{x+1}$ in the default viewing rectangle. There is something wrong with this graph. There appears to be a vertical line near the value $x = -1$. This line is **not** part of the graph of the function. It is called an **asymptote.** We can use the command **/) Pnts/Ln** to see that this line is not part of the graph.

Select **/) Pnts/Ln** and then **R) Replot** to redraw the graph of the function in the **/) Pnts/Ln** mode. The function will be replotted using only points without the connecting line segments. Figure 2.56 shows the resulting screen display.

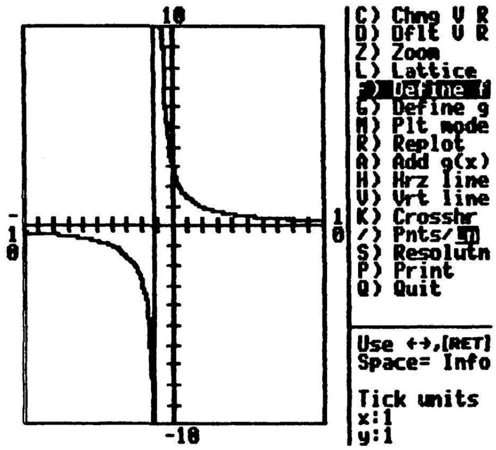


Figure 2.55

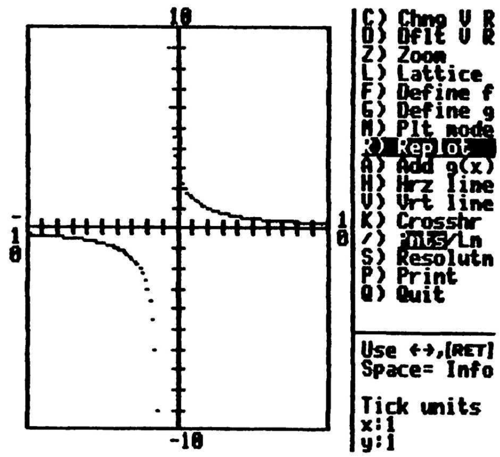


Figure 2.56

Now we can see that the vertical line in Figure 2.55 is **not** part of the graph. You may select the **S) Resolutn** command to increase the density of the points plotted in a graph. Drawing graphs with a computer is very fast, but you still have to be sure that you get a correct graph!

2.5 Apple II Conic Grapher

Most interactive commands for the *Conic Grapher* are the same as those for the *Function Grapher*. The default graph is $x^2 + y^2 - 9 = 0$, a circle with a radius of 3 centered at the origin.

E) Defn Eqs: You will see this command on the **commands menu** in place of **F) Define f** in the *Function Grapher.* This new command is used to enter conic equations and functions into the program. You may enter 2 different conic equations and 1 function in the program at once. Functions are entered in the same way as in *Function Grapher.* Conic equations must be entered in their general form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

where $A, B, C, D, E,$ and F are the coefficients of the terms of the general conic equations. These 6 coefficients must be entered in order. For example, suppose you want to graph an ellipse with

equation $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$. First rewrite the equation in the general form $9x^2 + 25y^2 - 225 = 0$. Notice that $A = 9, B = 0, C = 25, D = 0, E = 0$, and $F = -225$. After each coefficient is entered, press Return . For coefficients which are zero, simply press Return and the program will automatically enter the zero. Figure 2.57 shows the screen with the 6 coefficients for this ellipse entered and Figure 2.58 shows the graph of the ellipse.

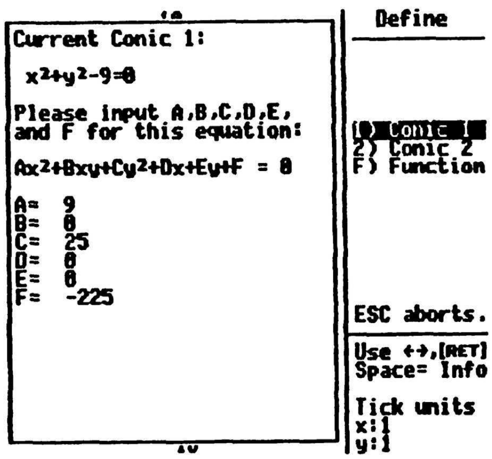


Figure 2.57: Enter the coefficients

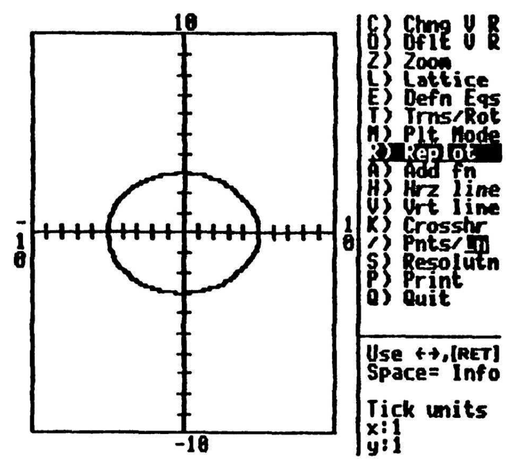


Figure 2.58: Graph of the ellipse

The graph is an ellipse centered at the origin with major axis of length 10 on the x-axis and minor axis of length 6 on the y-axis.

The **E) Defn Eqs** command is also used to enter the second conic equation or the function. The graph(s) which are automatically drawn on the screen depend on the **Plot Mode** settings chosen using the **M) Plot mode** command. You may choose to draw the graph of Conic # 1, Conic # 2, or the function, or any combination of these three relationships.

T) Trns/Rot: This command allows you to **translate or rotate** either conic equation you have defined. Translation of a conic graph is done by specifying horizontal and vertical translation factors for the center of the conic. For example, entering an x translation factor of 3 and a y translation factor of -4 for the ellipse defined above will redraw the graph of the ellipse centered at the point (3, -4). Figure 2.59 shows this translated graph. Any further translations of the graph are always relative to the original graph and not to a translated graph.

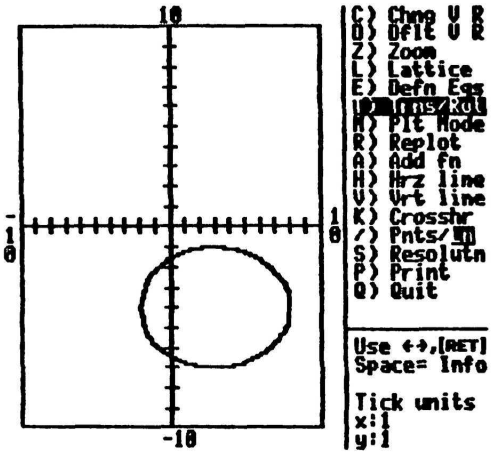


Figure 2.59: Translated ellipse

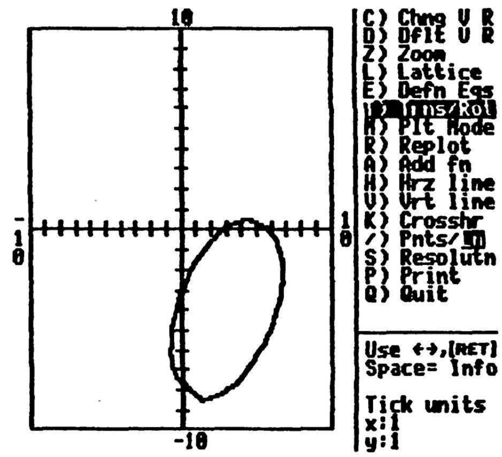


Figure 2.60: Rotated ellipse

This command also allows you to rotate any conic graph a specified number of radians about the origin. Positive values will rotate the graph counterclockwise and negative values will rotate the graph clockwise. As with translations, all rotations are relative to the original graph and not a rotated graph. However, once a graph is translated, rotation is done on the translated graph and vice versa. Figure 2.60 shows the graph of the translated ellipse defined above, rotated 1 radian counterclockwise.

Space Bar Functions: The [Space Bar] has two important functions in the *Conic Grapher*. While a graph is being plotted, pressing the [Space Bar] will pause the plotting process. Press any key to continue. At any other time, pressing the [Space Bar] will display a window showing the conic equations and functions entered into the program, and the current translation and rotation factors. Figure 2.61 shows the effect of pressing the [Space Bar] while the graph is being drawn and Figure 2.62 shows the information window for the above example.

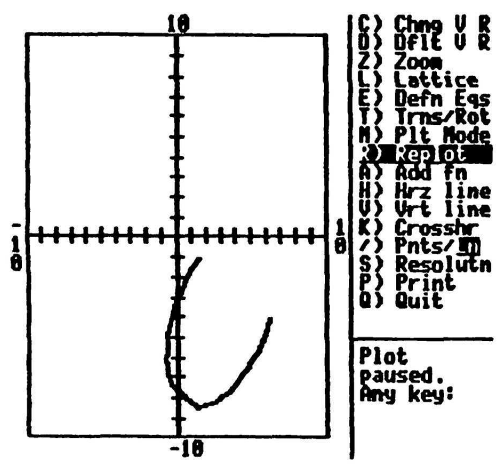


Figure 2.61: Pause the plot

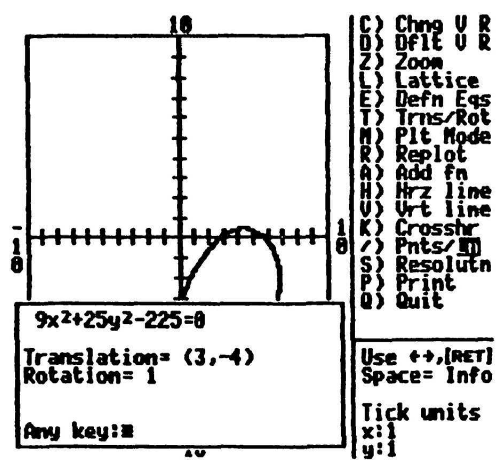


Figure 2.62: Information window

All other commands in the *Conic Grapher* operate in the same way as those explained for the *Function Grapher.*

2.6. Apple II Parametric Grapher

Most interactive commands for the *Parametric Grapher* are the same as those for the *Function Grapher*. The default screen shows the graphs of two sets of parametric equations:

$$x_1 = 75 - 30T \qquad y_1 = 30\sqrt{3}\,T - 16T^2$$
$$x_2 = 20\cos\frac{\pi T}{6} \qquad y_2 = 20 + 20\sin\frac{\pi T}{6}.$$

The range of the parameter T is $0 \le T \le 2$. Figure 2.63 shows the default screen; Figure 2.64 shows the equation information window activated by pressing the [Space Bar] . Pressing the [Space Bar] while the graphs are plotting will pause the plot and print out the value of the parameter, T, and the coordinates of the last point(s) of the graph(s) plotted.

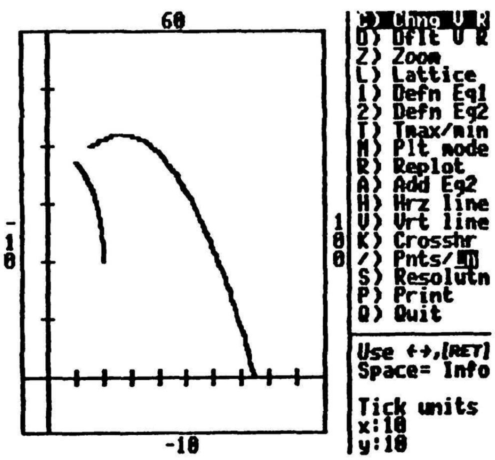


Figure 2.63: Default screen

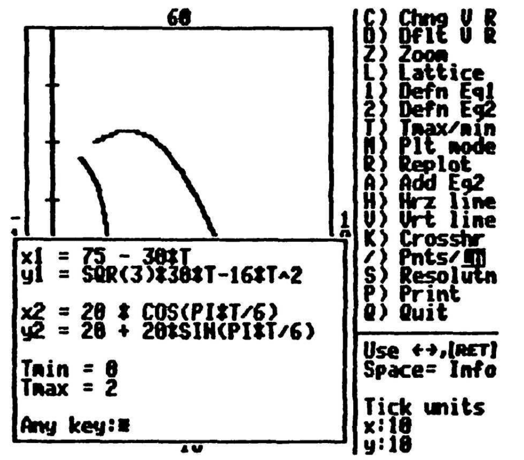


Figure 2.64: Equation information window

1) Defn Eq1 and **2) Defn Eq2**: These commands are used to enter the sets of parametric equations into the computer. Parametric equations are functions in terms of T which are used to evaluate the x- and y-coordinates of each point. For example, select **1) Defn Eq1** and enter the parametric equations $x_1 = \sin(3T)$ and $y_1 = \cos(5T)$. After the you enter the equations, the computer will ask for the range of T. Enter the values for **Tmin** and **Tmax**. For this problem enter **-PI** for **Tmin** and **PI** for **Tmax**. (The program recognizes the entry **PI** as π and enters an 8-decimal place approximation for the value.) Figure 2.65 shows the screen with the equations and the range of T entered. Figure 2.66 shows the resulting graph in the viewing rectangle [-1, 1] by [-1, 1].

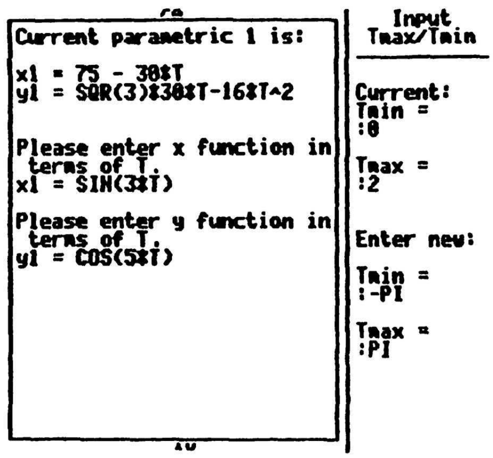


Figure 2.65: Enter equations & range of T

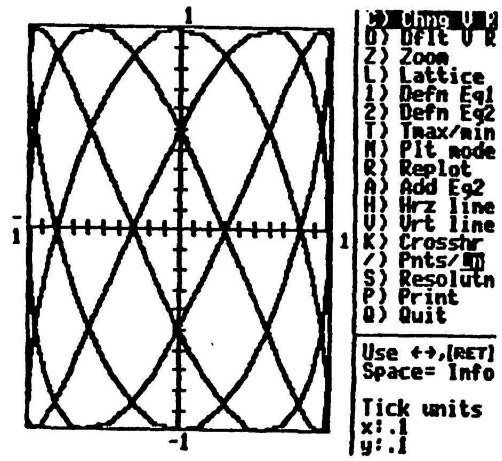


Figure 2.66: Resulting graph

You may enter one or two sets of equations and, depending on the **Plot Mode** setting (set by using the **M) Plt mode** command), either one or both of the equations can be drawn automatically. **Make sure you enter equations in terms of *T* and not *x* as in the *Function Grapher*.**

T) Tmax/min : This command allows you to choose a new maximum and minimum value for the parameter T of the function you are graphing. The procedure is identical to the last steps in the **1) Defn Eq1** command outlined above.

2.7 Apple II Polar Grapher

Most interactive commands for the *Polar Grapher* are the same as those for the *Function Grapher*. The default screen shows the graphs of $r(t) = 7 \sin(3T)$ with the value of T in the range $0 \le T \le \pi$.

1) Defn Eq1 and **2) Defn Eq2** : These commands are used to enter the polar equations into the computer. Unlike the *Parametric Grapher*, each polar graph has only one equation. For example, the polar equation $r_1 = 10 \cos(6T)$ is the only equation needed to define the graph (see Figure 2.67). Like the *Parametric Grapher*, you must enter a range for the parameter T in terms of a **Tmin** and a **Tmax** value. Figure 2.68 shows the screen with the information window visible. The value of **Tmax** was entered as **2 * PI**. The computer understands **PI** to be π and can do computations with **PI** within an input statement.

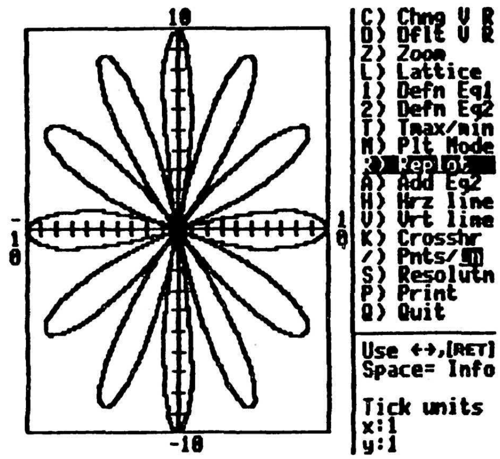


Figure 2.67: Graph of $r_1 = 10 \cos (6T)$

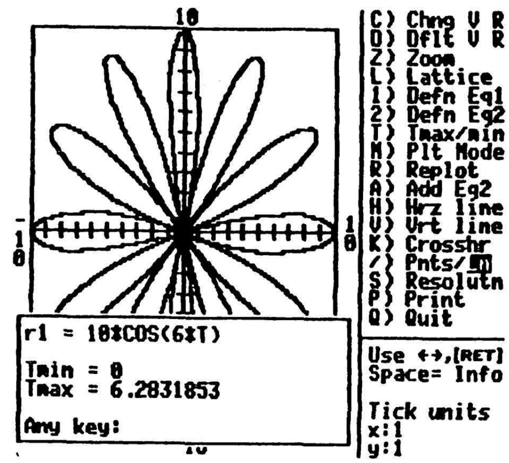


Figure 2.68: Information window visible

T) Tmax/min : This command allows you to choose a new maximum and minimum value for the parameter, *T*, of the function you are graphing. This parameter represents the angular rotation from the positive *x*-axis in the polar coordinates (r, Θ). Often the range of *T* will be in terms of π or multiples of π. Use the keyboard equivalent of **PI** for π or multiples of π like **2*PI** or **PI/2.**

K) Crosshr : In the *Polar Grapher* you will notice a slight difference in the way the crosshairs work. When you have placed the crosshairs on a point whose coordinates you wish to see, press the [Space Bar] to see the *x*- and *y*-coordinates of the point in the lower right corner of the screen as usual. You will be prompted to press any key. When you press a key, the values for *T* and *r* will be displayed. Finally, pressing any key again will return you to the crosshair menu.

S) Resolutn : When graphing polar equations, it will often be necessary to increase the resolution of the graph to get accurate pictures. Resolutions of 120 or 180 will give better results than the default value of 60.

Important Notes:

1. **PI** and **E** are acceptable constants for π and *e*.

2. Any BASIC language expression may be input for <u>any</u> numerical input value. For example, you may enter **PI/2, SIN(3),** or **ATN (E*3)** in a "GOTO" or "Zoom factor" input statement.

3. Pressing the [Esc] key gets you out of <u>anything</u>. If pressing [Esc] fails (it shouldn't ever), try pressing [Control] [Reset] at the same time. Pressing [Control] [Reset] returns to the **commands menu** and [Esc] returns to the active menu, such as the **Zoom menu** or the **Plot mode menu.**

2.8. Apple II 3D Surface Grapher

2.8.1 Introduction : A great deal is known about single variable function graphers. The use of such graphers and understanding about how they can enhance the teaching and learning of mathematics are on the rise. Much less is known about the use of surface graphers, that is, devices that produce a graph of a function of two variables. However, several things about graphing functions of two variables are very clear. Obtaining graphs by hand is a difficult task for both students and teachers. Students have a good bit of trouble visualizing in three dimensions. Teachers have a difficult time producing quick, accurate graphs of functions of two variables.

The three dimensional grapher described in this guide is designed to allow the user to obtain reasonably accurate graphs of functions of two variables. The user can obtain the graph for $a \le x \le b$, $c \le y \le d$, and $e \le z \le f$ and then choose an arbitrary point in three dimensional space from which to view the graph. Once the first graph is drawn the points are stored in an array so that the graph can be redrawn quickly from different views. The user can choose any point in the three dimensional space from which to view the graph. The resolution of a graph is under user control.

The three dimensional grapher allows the user to interactively explore the behavior of surfaces. Local maximum and minimum values of the functions of two variables can be investigated graphically. The grapher can help students deepen their understanding and intuition about functions of two variables. It can provide a geometric representation of problem situations to go along with an algebraic representation. The connections between these two representations can be explored and exploited to gain better understanding about problem situations.

The single most important feature of this graphing program is that virtually every aspect of this utility is interactive and under user control. This utility was designed to help teachers teach and students learn mathematics in an atmosphere where both are active partners in the educational experience.

2.8.2 Drawing and Viewing a 3D Graph . This section describes how the user chooses a region of three dimensional space in which to draw a graph of a function of two variables, and the way in which that graph can be viewed.

Definition: The set $\{ (x, y, z) \mid a \le x \le b,\ c \le y \le d,\ e \le z \le f \}$ is called the ***viewing box*** **[a, b] by [c, d] by [e, f]** where a = x-minimum, b = x-maximum, c = y-minimum, d = y-maximum, e = z-minimum, and f = z-maximum values of the viewing box.

Notice that the viewing box [a, b] by [c, d] by [e, f] is completely determined by the region $a \le x \le b$ by $c \le y \le d$ by $e \le z \le f$ of three dimensional space. The user can change the viewing box by selecting the **C) Chng V D** option from the **commands menu** and entering the new values corresponding to x-min, x-max, y-min, y-max, z-min, and z-max of the viewing box. Figure 2.69 shows the default function, ***f(x, y)* = sin(*y*)**, in the default viewing box of [-10, 10] by [-10, 10] by [-10, 10]. Figure 2.70 shows the screen for the **C) Chng V D** option with the new viewing domain [-5, 5] by [-4, 4] by [-6, 6] entered. Figure 2.71 shows the graph of ***f(x, y)* = sin(*y*)** redrawn in the new viewing box [-5,5] by [-4, 4] by [-6, 6].

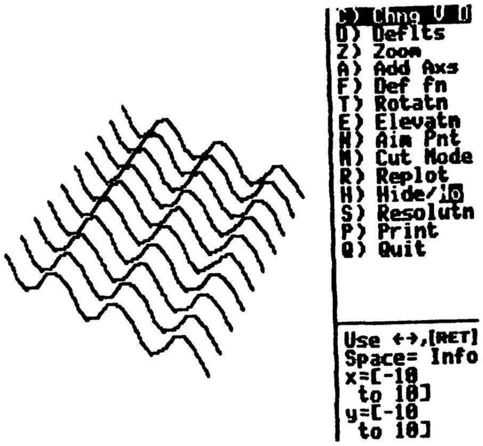


Figure 2.69: $f(x, y) = \sin(y)$

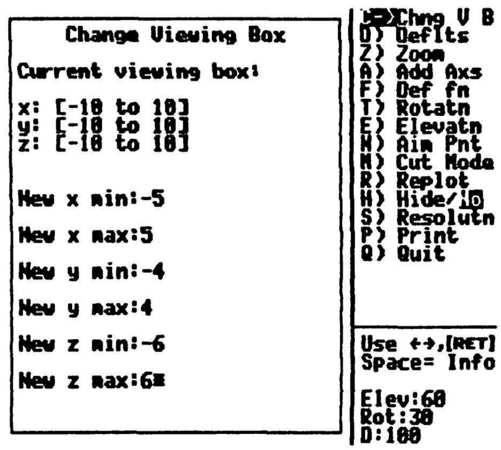


Figure 2.70: Change Viewing Box screen

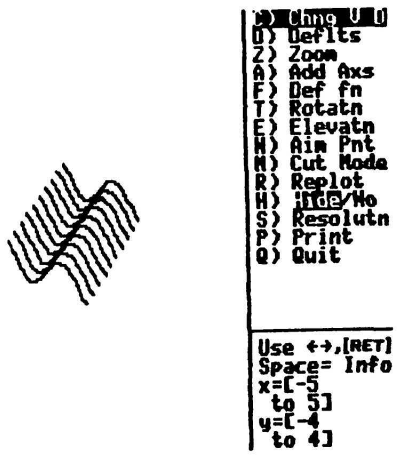


Figure 2.71: Graph of $f(x, y) = \sin(y)$ in [-5, 5] by [-4, 4] by [-6, 6]

Next, the user decides how to view the graph contained in the selected viewing box. Two points can be selected. The point at which the user places his/her "eye" is called the ***viewing point***. The point at which the view of the eye is directed is called the ***aiming point***. The default aiming point is (0, 0, 0), the origin of the three dimensional space. The aiming point can be changed by selecting **N) Aim Pnt** from the **commands menu** and inputting the rectangular coordinates of the point. The viewing point can be changed by entering the spherical coordinates (d, ϕ, θ) of the point. Figure 2.72 shows the relationship between the aiming point, the viewing point, and the xyz- axes.

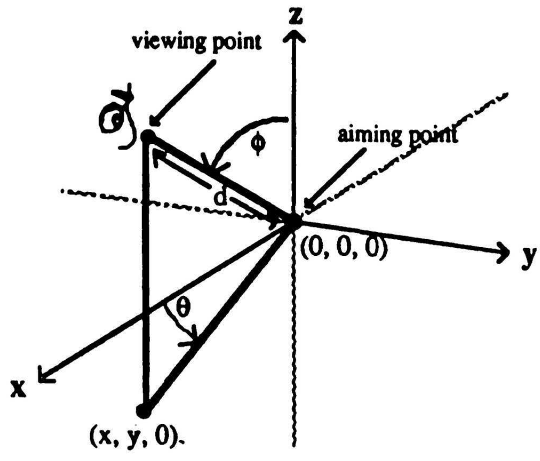


Figure 2.72: Orientation in 3D space

The spherical coordinate d of the viewing point represents the distance from the aiming point to the viewing point. This parameter is controlled by selecting **Z) Zoom** from the **commands menu.** The coordinate θ represents the angle that the plane perpendicular to the xy-axis containing the viewing point makes with the x-axis; the positive direction is counterclockwise. This parameter is controlled by selecting **T) Rotatn** from the **commands menu.**

The coordinate ϕ represents the angle the line through the aiming point and the viewing point makes with the z-axis. A value of $\phi = 0°$ means you are looking directly down on the z-axis. A value of $\phi = 90°$ means you are looking perpendicular to the z-axis, parallel to the xy-plane. Value of ϕ between 90° and 180° means you are viewing the graph from below the xy-plane. This parameter is controlled by selecting **E) Elevatn** from the **commands menu.** The default viewing point is (100, 60°, 30°).

The software draws the graph in a cone of vision determined by the aiming point and the viewing point. The viewing point is the vertex of the cone, and the line determined by the viewing point and the aiming point is the axis of the cone. The view is from the viewing point toward the aiming point. The option **H) Hide/No** allows the user to view the graph with or without hidden lines. That is, if the hidden lines option is on, then the user will not see the portions of the surface that should be hidden from view by other portions of the surface. Basically, the graph that the user sees is the intersection of the cone of vision with the user selected viewing box. By changing the aiming point, viewing box, and the viewing point, the user can view *any* portion of a surface with a high degree of resolution.

This drawing and viewing feature of the software literally allows the user to move around and view the surface as if in an airplane. You can move closer or further away, and view the graph above or below by careful selection of elevation, rotation, and distance (see Figure 2.73).

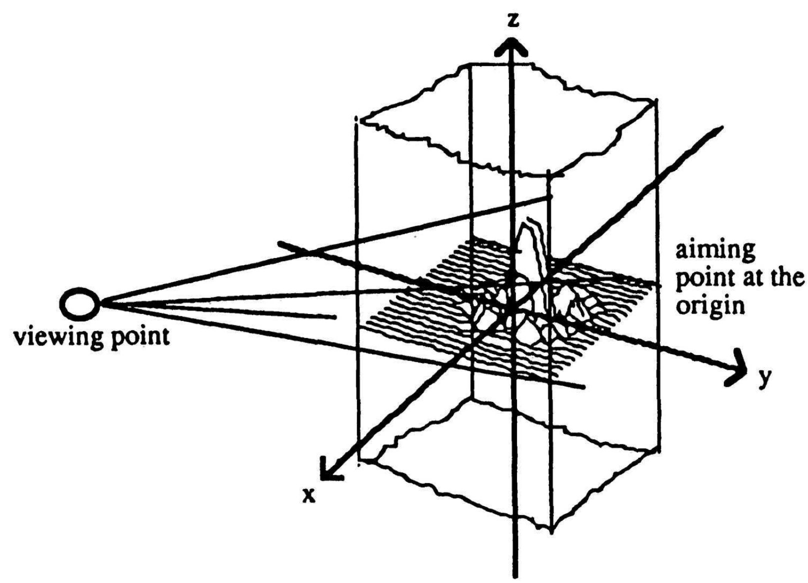


Figure 2.73: Orientation of the viewing point, aiming point and the graph

2.8.3 Interactive Menu Commands for the 3D Surface Grapher

The following is a detailed explanation of the options on the **commands menu.**

C) Chng V B (Change viewing box): This command allows the user to change the dimensions of the viewing box. The default viewing box is [-10, 10] by [-10, 10] by [-10, 10]. Figure 2.70 shows the screen with the new values [-5, 5] by [-4, 4] by [-6, 6] entered.

D) Deflts (Defaults): This option replots the current graph with the default aiming point of (0, 0, 0), the default viewing point of (100, 60°, 30°), in the default viewing box of [-10, 10] by [-10, 10] by [-10, 10], and using only x-axis cuts.

Z) Zoom : This option changes the distance from the viewing point to the aiming point. A distance of 100 (the default setting) causes the viewing box [-10, 10] by [-10, 10] by [-10, 10] to approximately "fill" the computer screen. Figure 2.74 shows the default function, $f(x, y) = sin\ (y)$ drawn with a zoom factor of 200 and Figure 2.75 shows the same function drawn with a zoom factor of 50.

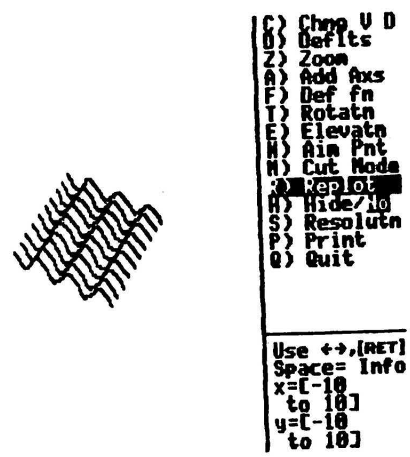


Figure 2.74: Zoom factor of 200

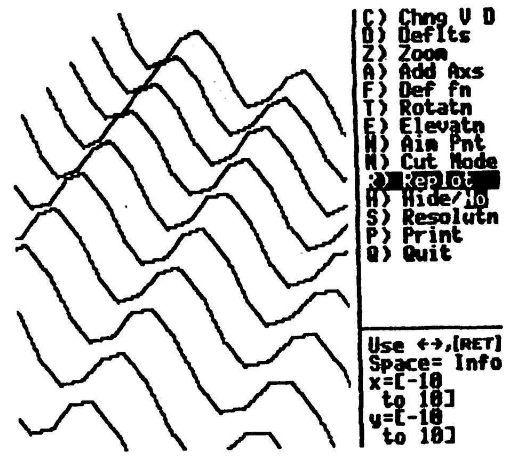


Figure 2.75: Zoom factor of 50

Changing the zoom factor does not change any of the other parameters of the graph relating to aiming point or viewing point. Select **R) Replot** to redraw the graph.

A) Add Axs (Add Axes): This option allows the user to superimpose the *x*-, *y*-, and z-axes on the graph. Figure 2.76 shows the default graph of *f*(*x*, *y*) = sin (*y*) with the axes in place. To remove the axes, select **R) Replot.**

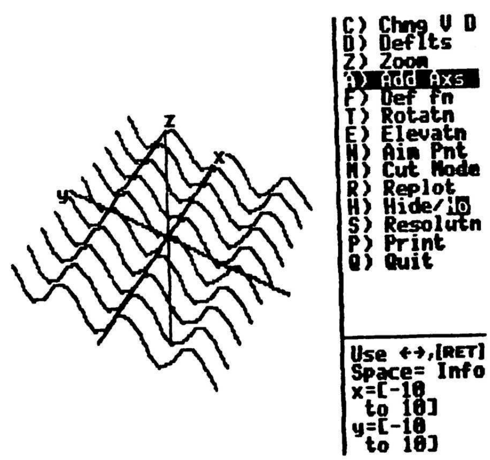


Figure 2.76: Axes superimposed on the graph

F) Def fn (Define function): This option is used to enter a new function. Functions are entered in terms of *x* and *y* using standard AppleSoft BASIC language symbols. The constants *e* and π are entered as [E] and [P][I] respectively. Special keying instructions are needed to enter built-in functions or to obtain correct graphs of some special functions.

Here are some examples.

1) The absolute value function $|x + y|$ is entered as **ABS (X + Y)**.
2) The natural logarithm function $ln\ y$ is entered as **LOG (Y)**. Note: $y = \log_b (y)$ can be graphed for any base b by entering **LOG (Y) / LOG (B)**.
3) The exponential function e^{x+y} is entered as **EXP (X + Y)**.
4) The root function $\sqrt[n]{x}$ for n odd is entered as **ABS (X) / X * ABS (X) ^ (1 / N)**.
5) The greatest integer function $[x]$ is entered as **INT (X)**.
6) The signum function is entered as **SGN (Y)**.
7) The square root function is entered as **SQR (X - Y)**.

T) Rotatn (Rotation): This option is used to change the angle, θ, that the plane perpendicular to the xy-axis containing the viewing point makes with the x-axis (see Figure 2.72). Positive values for θ indicate a counterclockwise rotation viewed from the top; negative values indicate a clockwise rotation. Figure 2.77 shows the default function at the default rotation of $\theta = 30°$ and Figure 2.78 show the same function rotated $\theta = 15°$.

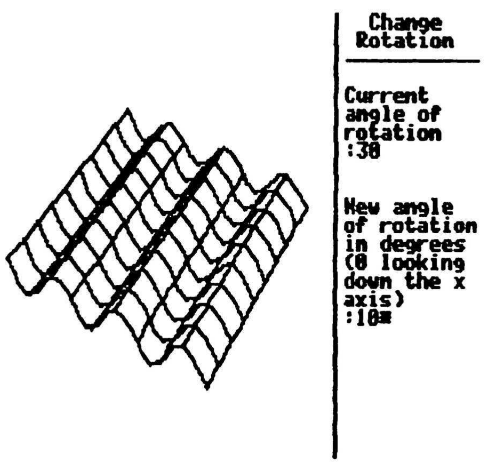


Figure 2.77: Default graph

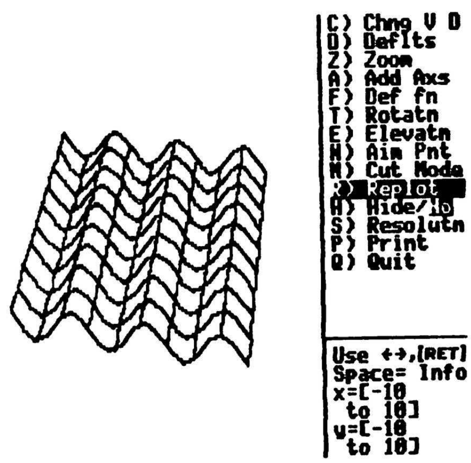


Figure 2.78: Rotation of 15°

E) Elevatn (Elevation): This option controls the angle that the line from the viewing point to the aiming point makes with the z-axis (see Figure 2.72). A value of $\phi = 0°$ means you are looking directly down the z-axis from above. A value of $\phi = 90°$ means you are looking perpendicular to the z-axis, parallel to the xy-plane. Values of $0° \le \phi \le 90°$ mean you are viewing the graph from above the xy-plane. Values of $90° \le \phi \le 180°$ mean you are viewing the graph from below the xy-plane. The default value is $\phi = 60°$. Figure 2.79 shows the default graph with an elevation of $\phi = 75°$.

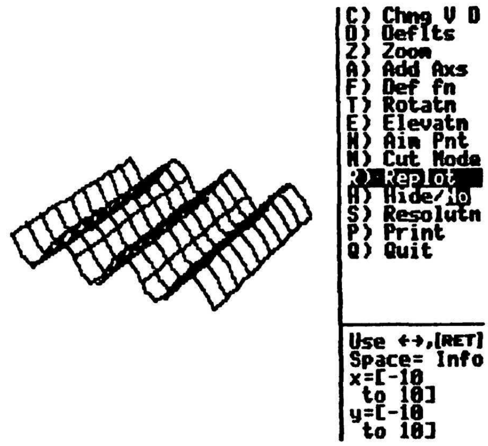


Figure 2.79: Elevation of ϕ = 75°

N) Aim Pnt (Aim Point): This option controls the location of the aiming point. The default aiming point is (0, 0, 0), the origin of the three dimensional space. The aiming point can be changed by inputting the rectangular coordinates of the point you wish to be the center of the viewing cone.

M) Cut Mode : Select this option to choose the axis through which the points of the graph will be plotted. The default cut mode is **x-cut only.** This means that the graph will be plotted based on values along the x-axis. By selecting **1) y then x** from the **Cut Mode menu,** and **R) Replot** the graph will be drawn with both the x- and y-cuts (see Figure 2.80).

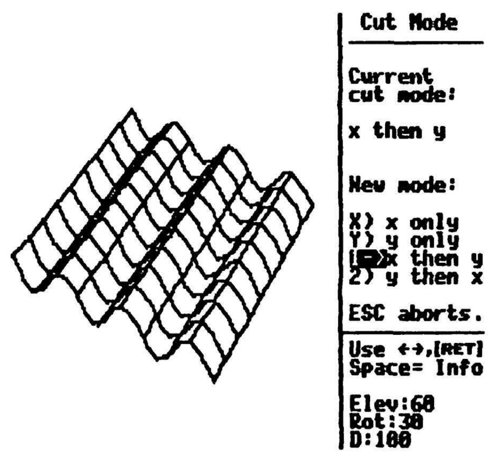


Figure 2.80: Cut Mode menu and default graph plotted with x- and y-cuts

Using both x- and y-cuts gives a better representation of the 3D surface. However, using this drawing mode requires much more time for computation of points and drawing the graph.

R) Replot : Use this option to redraw the graph after changing any of the parameters.

H) Hide/No (Hidden lines/ No hidden lines): This option allows the user to plot a graph with or without hidden lines. That is, if the hidden lines option is on, then the user will not see the portions of the surface that should be hidden from view by other portions of the surface. When the hidden lines option is off, all computed points will be plotted. Figure 2.81 shows the default graph with an elevation of $\phi = 75°$ plotted with the hidden lines option turned on. Notice that some of the surface is behind other portions giving a three dimensional look to the two dimensional representation.

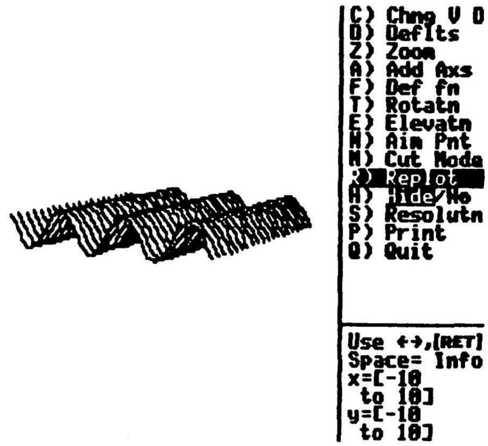


Figure 2.81: Hidden lines option turned on

S) Resolutn (Resolution): This option controls the number of sections and the number of points per section which are computed to draw the graph. The default graph is drawn with 10 x-cut sections of 20 points each. This means that 10 x 20 = 200 individual points are computed and then plotted on the screen. Increasing the number of sections and/or the number of points per section draws a graph with better resolution, but require more time. For example, choosing the maximum number of sections (20) and the maximum number of points per section (70) in both the x and y direction will require the computation and plotting of 2800 individual points (as compared to 200 for the default graph). Figure 2.82 shows the default graph drawn with the maximum number of sections and points.

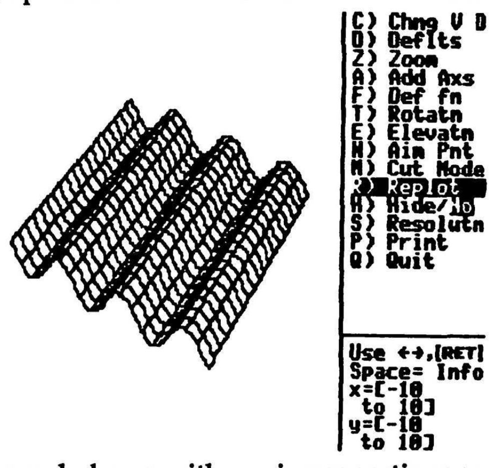


Figure 2.82: Default graph drawn with maximum sections and points per section

P) Print : This option prints the current graph to the printer.

Q) Quit :This option ends the current session and returns to the **Main Menu.**

2.8.4 Helpful Hints

(1) Press the [Esc] key to abort the computation of points or the plotting of the graph. The [Esc] key feature is useful when you select a high resolution and want to change it before waiting for the complete array of values to be computed or the graph to be drawn.

(2) The [Space Bar] has two functions: 1) Pressing the [Space Bar] while a graph is being plotted will cause the plot to pause and the coordinates of the last point plotted to appear in the window at the lower right of the screen. Press any key to continue the plot (see Figure 2.83). 2) Pressing the [Space Bar] at any other time will show an information window listing the current function, elevation, rotation, distance, aiming point, and the number of sections and points per section used to draw the graph. Figure 2.84 shows the information window for the default function.

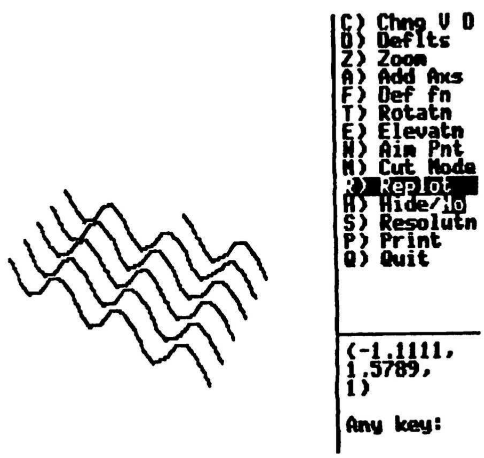


Figure 2.83: Graph paused

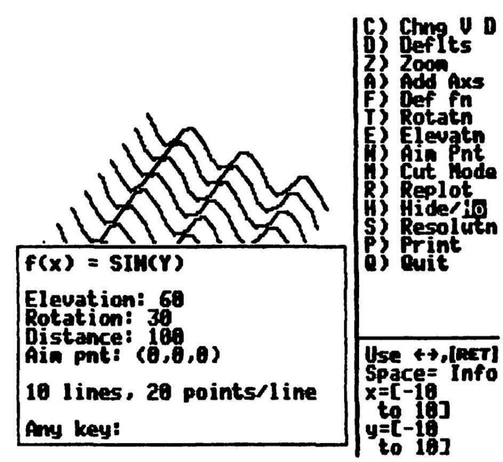


Figure 2.84: Information window

(3) Because of the increased plotting time, increase resolution only when you desire a "nice" plot.

(4) Use the **Z) Zoom** option to change the cone of vision. Changing the distance *d* allows you to see more, or less, of the graph contained in the viewing box.

(5) To zoom-in on features away from the center (aiming point) of the current graph, you may need to change both the viewing box and the aiming point. However, do this in low resolution until you are sure you have selected the viewing box and aiming point that displays the desired features.

Chapter 3

Master Grapher Version 1.0 for the IBM ^{R}

3.1 MASTER GRAPHER START UP

IMPORTANT: Do NOT write protect either the *MASTER GRAPHER* , or *3D GRAPHER* disk; the program writes to the disk!!!!

Start Up: Boot the unit with DOS 2.1 (or greater) in drive A, then insert the *Master Grapher* disk into drive B. The program should run on a machine with only 256K of memory if it is run under DOS 2.X, however it will not run under DOS 3.X on the same machine because of the amount of memory required by DOS 3.X.

Screen Dumps: Type *Graphics* after the A > prompt if you want to use the Shift PrtSc key for screen dumps of your graphs. To access drive B, type b : and press the return key. To run the graphing program, type *MASTER* and press the return key.

Hercules Graphics Card: If you have a Hercules graphics card or compatible you will need to follow the following instructions instead of the preceding. To access drive B, type b : and press the return key. Type *PRTSCR* after the B > prompt if you want to use the Shift PrtSc key for screen dumps of your graphs. To run the graphing program, type *MASTER-H* and press the return key.

Main Menu: Information about the graphing program will appear on the screen. Press any key to go to the *main menu*. The *main menu* will give a list of the four different graphing programs that are available for your use.

3-D Grapher: *Master Grapher* includes *3-D Grapher*, a graphing utility for functions of *two* variables. The *3-D Grapher* utility is described in section 3.7 - 3.10 on pages 50-54.

Main Menu
Type the appropriate key
1) Function Grapher
2) Conic Grapher
3) Parametric Grapher
4) Polar Grapher
5) Help
6) Exit

3.2 FUNCTION GRAPHER

When you are ready to enter the Function Grapher, select 1. There will be a slight delay while the function grapher program is loaded. The initial screen (except for the function graphed and this viewing rectangle) will look like the one shown in the figure 3.1.

The interactive commands available for your use will be in the menu on the right side of the screen under the Function Menu. In the lower right corner are listed the distance between horizontal scale marks (HS) and the distance between the vertical scale marks (VS), as well as the translation factors (XT and YT), and the rotation factors (R).

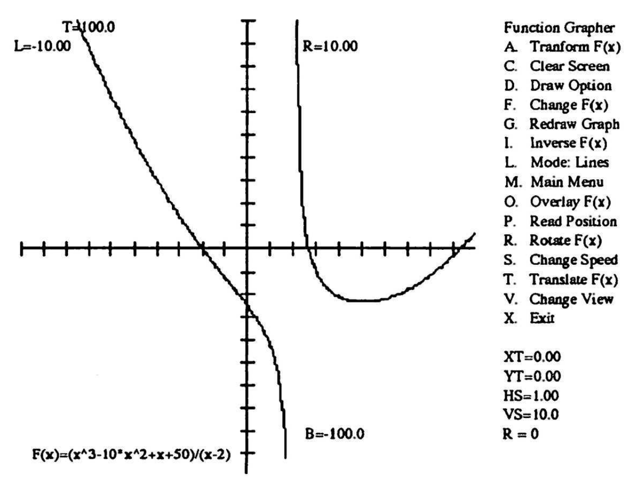


Figure 3.1

3.2.1 Change F(x): Select [F] to change the function to be graphed. You will see a menu with 10 options. Pressing [1] through [8] will toggle between *Not Displayed* and *Displayed*. Only the *Displayed* function(s) will be plotted. To change a function from Displayed to Not Displayed (or vice versa) type the index of the function whose status you wish to change. Thus, you can *predefine* and plot up to 8 functions in this manner. Select [9] to return to the default menu.

Select [0] to change any one of the eight functions listed. When you press [0], you will then be asked which function index you wish to change. (There will be no prompt). You will now be able to edit the function you specified; to do this you can use keys in the table below. Edit the desired equation you will want to graph (in terms of x in the function and conic grapher and in terms of t in the polar and parametric graphers), then press [return].

Editing Keys

[→]	Moves the cursor to the right one character. If you are at the end of the equation this has no effect.
[←]	Moves the cursor to the left one character. If you are at the beginning of the equation this has no effect.
[backspace]	Has the same effect as the [←] key.
[Del]	Deletes the character at the current cursor position.
[End]	Moves the cursor to the end of the equation.
[Esc]	Deletes the edited equation and sets the cursor to the first position.
[Home]	Moves the cursor to the end of the equation.
[Ins]	Changes insert mode from overstrike to insert. Pressing it again reverses the process.
[Return]	Truncates the equation where the cursor is and exits edit mode. If an error is made in the equation it will revert back to the original equation.

Use standard BASIC syntax to enter the desired equation. To enter $x^4 - 3x^2 + 15$ for example you would enter [x] [∧] [4] [-] [3] [*] [x] [∧] [2] [+] [1] [5]. The constants e and π are entered as [e] and [p] [i], respectively. Special keying instructions are needed to enter built-in functions or to obtain correct graphs of some special functions. Here is a list of the special functions.

Special Symbols & Built-in Functions:

1. $+$ is addition.
2. $-$ is subtraction.
3. $*$ is multiplication.
4. $/$ is division.
5. $\wedge$ is x^a.
6. $\backslash$ is Integer Division.
7. ABS($x-2$) is $|\,x-2\,|$.
8. CEIL(x) $= [\![x+1]\!]$.
9. EXP(x) is e^x.
10. FIX(x) is FLOOR if $x > 0$, and CEIL if $x < 0$.
11. FLOOR(x) $= [\![x]\!]$.
12. INT(x) is the greatest integer function .
13. LOG(x) is $\ln x$.
14. LOG10(x) is $\log_{10}(x)$.
15. LOG2(x) is $\log_2(x)$.
16. ROUND(x) rounds to the nearest integer.
17. SGN(x) is the signum function.
18. SQR($x+6$) is $\sqrt{x+6}$.
19. SIN(x) is $\sin\ x$. All other trigonometric functions are entered in the same manner (e.g. arctan x is ARCTAN(x), $\cosh x$ is COSH(x), etc.). Here is a list of all the trigonometric functions supported: arccos, arccosh, arccot, arccoth, arccsc, arccsch, arcsec, arcsech, arcsin, arcsinh, arctan, arctanh, cos, cosh, cot, coth, csc, csch, sec, sech, sin, sinh, tan, tanh.

Special Functions:

20. LOGB(x, a) is $\log_a(x)$.
21. ROOT(x, a) is $x^{1/a}$. Note: you can enter $x^{1/a}$ as $x \wedge (1/a)$, but you will only get the portion of the graph in the first quadrant.
22. POWER(x, a) is x^a.

3.2.2 Change View: Selecting [V] will display the menu to the right. You may have changed the speed, the plot mode or a variety of different commands that do not cause the graph to be immediately replotted; the function grapher incorporates those changes when it is redrawn or the viewing rectangle is changed. Suppose we want to view the graph of

$$f(x) = \frac{x^3 - 10x^2 + x + 50}{x - 2}$$

in the viewing rectangle $[-1, 1]$ by $[-1, 1]$. To do this enter f by selecting [F] and proceeding as detailed in the "Change F(x)" Section 3.2.1 (page 43), then select [V]. You can now choose the method you want to change the viewing rectangle.

View Menu

0. **Zoom In**
1. **Zoom In (Point)**
2. **Zoom Out**
3. **Zoom Out (Point)**
4. **Set Zoom Factor**
5. **Set Window**
6. **Default Window**
7. **Last Window**
8. **X-Scale: 1**
9. **Previous Menu**

0. Zoom-In: To set your own area for the new viewing rectangle, select [0]. Place the pointer at the desired corner of the zoom-in rectangle, then press the space bar to set the corner. Now use the ARROW keys to draw the desired viewing rectangle on the screen. Press the space bar to obtain a new plot. The arrow keys are on the numeric key pad. To move left, type [←] or [L]; to move right, type [→] or [R]; to move up, type [↑] or [U]; to move down, type [↓] or [D]. This time you will be moving the cursor around to locate a corner of the new viewing rectangle. Position the cursor as you did the last time. Now, use **only** the arrow keys to move toward the corner opposite the one you just marked for the new viewing rectangle. As you move in one direction, a straight line forms. Moving perpendicular to the initial direction you chose will cause a rectangle to form. The one corner you selected initially will remain fixed and you will be moving the opposite corner around. When you are satisfied with the location of the opposite corner of the box, press the space bar. You will now see the function plotted in the new viewing rectangle.

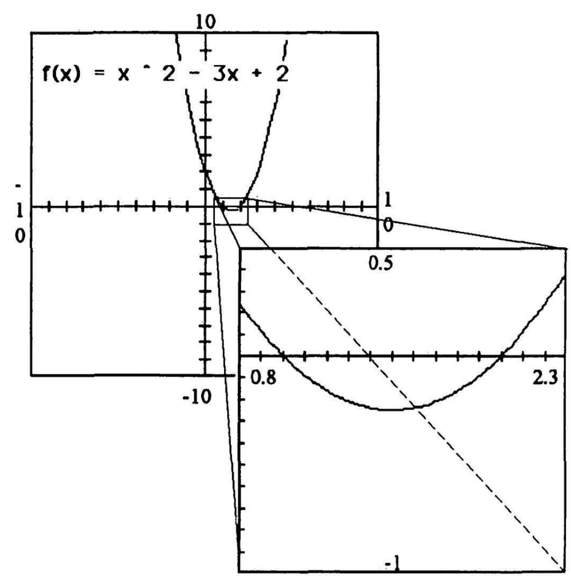


Figure 3.2

An illustration of the effect that "zoom-in" can have on a given view appears in Figure 3.2. The function $f(x) = x^2 - 3x + 2$ was graphed in the standard viewing rectangle, then graphed again in the $[0.8, 2.3]$ by $[-1, 0.5]$ viewing rectangle. The behavior of the graph between $x = 1$ and $x = 2$ is much more obvious in the zoom-in view.

1. Zoom In (Point): This command is useful when you want a close-up or magnified view of a particular section of a graph around a particular point. Select [1]. Use the Arrow keys or U, D, L, R to move the cursor. Type "S" to change the speed of the cursor's movement. Type any other key to set the

position. Once again you will use the arrow keys or the U, D, L, and R keys to move the cursor around on the screen. Use these keys to move the cursor in the current viewing rectangle to a point that you want to be the center of a new viewing rectangle. (Note: initially the cursor will be centered on the origin.) When you are satisfied with a location, press any key. The function will be replotted in a new viewing rectangle with the point you selected at the center of the new viewing rectangle. The *size* of the viewing rectangle will be determined by the zoom factor settings (key 4).

2. Zoom Out: When you select [2], the graph will be immediately redrawn in a viewing rectangle expanded by the zoom factors you set with key [4] (or the default values of 10 for x and 10 for y).

3. Zoom Out (Point): When you select [3], you will be asked to move the cursor to the location about which you wish the "zoom-out" to be centered in exactly the way explained above in *Key 1, Zoom In (Point)*. Once again the zoom factor will be determined by the default factors (both 10) or the factors you set using key [4].

4. Set Zoom Factors: This command sets the horizontal (x) and vertical (y) zoom factors. When you select [4] you will see the old setting for x and y zoom factors and be prompted to enter new ones. This can be done by typing in any real number or algebraic expression. Assume the current viewing rectangle is [L, R] by [B, T]. Enter the value you wish L - R and B - T to be multiplied by. Values greater than one cause the horizontal size of the rectangle to increase (zoom–out), and values less than one cause the horizontal or vertical size of the rectangle to decrease (zoom–in). The change in size of the viewing rectangle is symmetric about the center of the rectangle. Sometimes you will want the zoom factors on the x-axis and y-axis to be different. Note: You can "zoom–in" using "zoom–out" by selecting zoom factors that are less than one.

5. Set Window: To change the viewing rectangle, press [5]. The message screen will clear and you will see the current viewing rectangle settings where L, R, B, T are respectively left, right, bottom, and top of the viewing rectangle. It will now prompt you to enter the new parameters, starting with L. Type the real number or algebraic expression you prefer and press [`return`]. Next, you will enter the R, B, and T values. Answer each prompt with the desired real number or algebraic expression and press [`return`]. When [`return`] is pressed after entering T, the screen will be redrawn in the specified viewing rectangle and the program will return to the "View Menu".

6. Default Window: When you select [6], the displayed function(s) will be replotted in the default viewing rectangle ($[-10, 10]$ by $[-10, 10]$), speed (100 for the function grapher, 50 for the conic grapher and parametric grapher, and 200 for the polar grapher), mode (lines), rotation ($0°$), and translation $(1, 1)$. After the screen is redrawn you will be returned to the "View Menu".

7. Last Window: When you select [7], the displayed function(s) will be replotted in the last viewing rectangle. However speed, mode, rotation, and translation will remain unaltered from the current settings. After the screen is redrawn you will be returned to the "View Menu".

8. X-Scale: When you select [8], the distance of the tick marks will be changed on the x-axis. If X–Scale is in units of $\frac{\pi}{2}$ the distance between tick marks will be in a factor of units of either $\frac{\pi}{2}$ or $\frac{\pi}{4}$. If X–Scale is in units of 1 the distance between tick marks will be in a factor of units of either 1 or 0.5. The graph will be redrawn immediately with the appropriate x-scale. Note: This option is available only in the function grapher.

9. Previous Menu: When you select [9], this will return you to the grapher menu. Note: This option is [8] on all the other graphers.

3.2.3 Change Speed: To change the plotting speed, press [S]. You will now see the current plotting speed, the minimum and maximum allowable plotting speeds, and a prompt to enter a new speed. Enter the speed you want by typing any real number or algebraic expression with terms pi, e, or some other constants and then press [return]. The program will evaluate the expression and enter the resulting value as the new plotting speed. (You may wish to experiment with various settings until you find the one you prefer.) The default speed is a good compromise between speed and resolution.

3.2.4 Mode: Lines or Points: Select [L] to choose one of two plotting modes. One plots only the points evaluated for a particular function, the second connects each consecutive pair of points with a line segment. This is the default mode.

3.2.5 Rotate F(x): Select [R] to rotate the function about the origin. When you select [R], you will need to enter the counterclockwise rotation angle in degrees. This can be either a real number or an algebraic expression like "2*180/pi". A positive input will produce a counterclockwise rotation and a negative input will be clockwise.

3.2.6 Translate F(x): Select [T] to draw a graph of the function $y = f(x)$ translated H units horizontally and V units vertically. After selecting [T], enter the amount of horizontal translation and press [return]. Then enter the amount of vertical translation and press [return]. Nothing will appear to happen. To see the effect of the translation, you will need to redraw the graph [G]. You will now have graphed the *displayed* function with the translation factors applied. If you wish to view both the original function and the translated function, select [O] Overlay F(x) and choose the index of the original function and then [0] W/O Rot. & Trans. In this manner you can get both the function and its translation in the same viewing rectangle.

3.2.7 Redraw Graph: Redraws the graph with the current settings.

3.2.8 Overlay F(x): When you select [O] you will need to select an index function to overlay and then press [return]. For example, if you choose 6, then the sixth function in the function menu will be plotted on the same screen with the function(s) already plotted. Once the index is selected, you will need to tell the program if you want the function rotated and translated or not. If there are non-zero values in the XT, YT, and R categories in the lower right hand corner of the display screen, then choosing [1] and pressing [return] will have the selected function plotted *with* those rotation and translations applied. Choosing [0] will have the function plotted *without* any rotation or translation.

3.2.9 Clear Screen: Select [C] to clear the graphing screen.

3.2.10 Draw Option: Select [D]. A menu like the one to the right will appear with a list of options. Use [1] through [4] to approximate the coordinates of a point on a graph. To approximate the y-coordinate, a horizontal line can be specified or moved up and down (use the [↑] or [U] to move the line up, the [↓] or [D] to move the line down, and [S] to change the rate of movement of the line) until it is fixed in position by pressing any other key. The y-coordinate of each point on the horizontal line will be displayed. The x–coordinate can be found in a like manner using a vertical line (use the [→] or [R] and the [←] or [L] to move it right and left). Select [5] to overlay a lattice (an array of dots) in the viewing rectangle. The *distance* between these dots will be given by the HS (horizontal scale) and VS (vertical scale) values found in the lower right–hand corner of the screen. This command will be helpful to estimate the coordinates of a point on a graph. For

Draw Option
1. **Moving V. Line**
2. **Specified V. Line**
3. **Moving H. Line**
4. **Specified H. Line**
5. **Grid**
6. **Previous Menu**

example, zoom in on some area of the current graph that is of interest to you. Select [D] [5] to obtain a lattice in the viewing rectangle. Use the HS and VS values to read the coordinates of the point you selected.

3.2.11 Read Position: The next option [P] can also be used to approximate the coordinates of any point in the current viewing rectangle. When you select [P], a cursor will appear in the middle of the viewing rectangle. This cursor can be moved by using the arrow keys or pressing [U], [D], [L], [R]. To change the rate of movement of the cursor, type [S]. The speed control cycles from slow to fast so you may have to experiment with the speed each time you use it. When the cursor is at the desired location, type any other key. In the lower right–hand corner you will now see the location of the cursor in the viewing rectangle.

3.2.12 Inverse F(x): If you wish to view the inverse relation (y, x) where $y = f(x)$ of any function in the function menu, select [I]. Then select the index of the function you wish to invert and press [return], and the inverse *relation* will be overlayed immediately.

3.2.13 Transform F(x): This command allows you to draw the graph of $y = \mathrm{A}\, f\,(\mathrm{B}\, x\ +\ \mathrm{C}) + \mathrm{D}$, by specifying f, A, B, C, and D. Select [A]; you will then be asked to enter the index of the function f you wish to transform. Next you will be asked to enter the parameters A, B, C, D. You may enter any real number or algebraic expression (for example, "pi*2"). The function $y = \mathrm{A}\, f\,(\mathrm{B}\, x\ +\ \mathrm{C}) + \mathrm{D}$ will be plotted immediately.

3.2.14 Main Menu: Select [M] to return to the main menu.

3.2.15 Exit: Select [X] to return to the DOS operating system.

3.3 CONIC GRAPHER

When you are ready to enter the Conic Grapher, select [2] from the Main Menu. There will be a slight delay while the conic grapher program is loaded. The initial screen (except for the conic equation graphed) will look like the one shown in figure 3.3.

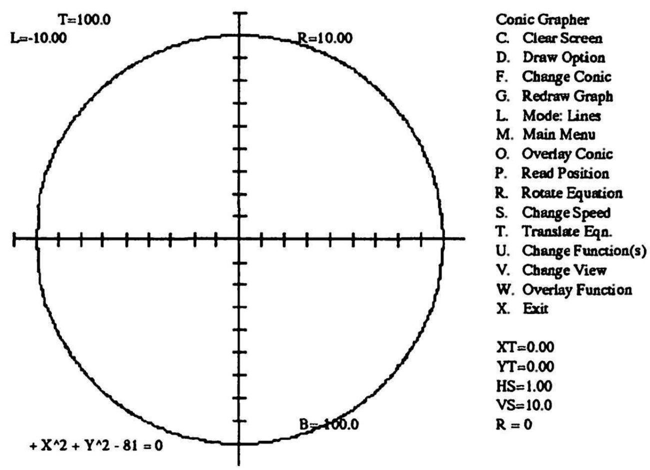


Figure 3.3

Notice the changes in the conic grapher menu versus the function grapher menu. The "A. Transform F(x)" and "I. Inverse F(x)" options have been removed. The "F. Change F(x)" is now "F. Change Conic" and the "O. Overlay F(x)" is now "O. Overlay Conic". Notice the "F(x)" in rotate and translate has been changed to equation. Finally, two new options have been added: "U. Change Function(s)" and "W. Overlay Function(s)". In the following sections we will only discuss these features as all the others have not changed. Note: rotate and translate have not been changed; only the wording in the menu has been changed.

3.3.1 Change Conic Pressing [F] will clear the viewing rectangle and you will see a list of 8 *conic equations.* Selecting any of 1-8 will allow you to toggle between *Displayed* and *Not Displayed.* There must always be at least *one* conic equation displayed. Selecting [9] will take you back to the conic grapher menu.

Selecting `O` will allow you to change a conic equation. When you press `O` you will be prompted to enter the conic index you wish to change. After entering the index you will be prompted to enter the parameters A, B, C, D, E, and F in that order. These parameters are the constants in the following equation: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. The constants may be entered as any real number or algebraic expression.

3.3.2 Overlay Conic Pressing `O` will allow you to overlay a *conic equation* in the same manner you did in section 3.2.8.

3.3.3 Change Function(s) Pressing `U` wil allow you to change or display any of the eight *functions* in the same manner you did in section 3.2.1.

3.3.2 Overlay Function Pressing `W` will allow you to overlay one of the eight *functions* in the same manner you did in section 3.2.8.

3.4 PARAMETRIC GRAPHER

When you are ready to enter the parametric grapher, select `3` from the Main Menu. There will be a slight delay while the parametric grapher program is loaded. The initial screen (except for the parametric equation graphed) will look like the one shown in figure 3.4.

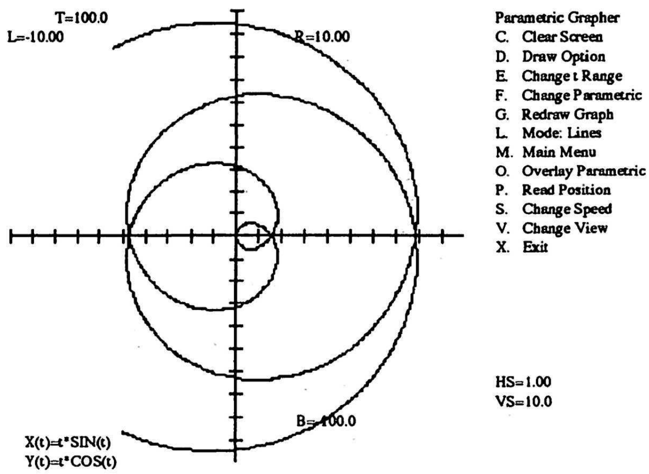


Figure 3.4

Notice the changes in the parametric grapher menu versus the function grapher menu. The "Transform F(x)", "Inverse F(x)", "Rotate Equation" , and "Translate Eqn." options have been removed. The "F. Change F(x)" is now "F. Change Parametric" and the "O. Overlay F(x)" is now "O. Overlay Parametric". Finally, a new option has been added: "E. Change t Range". In the following sections we will only discuss these features as all the others have not changed. This grapher will not plot the equations in the order of appearance in the "Change Equation" menu, but instead it plots the equations simultaneously so you can see the graphs drawn for the same t.

3.4.1 Change Parametric Select `F` to change the parametric equation to be graphed. You will see a menu with six options. Pressing `1` through `4` will toggle between *Not Displayed* and *Displayed.* Only the *Displayed* parametric equation(s) will be plotted. To change a parametric equation from Displayed to Not Displayed (or vice versa), type the index of the function whose status you wish to change. Thus, you can *predefine* and plot up to four parametric equations in this manner. Select `5` to return to the default menu.

Select `0` to change any one of the four parametric equations listed. When you press `0`, you will then be asked which function index you wish to change (there will be no prompt). You will now be able to edit the function you specified. To do this you can use the editing keys described in section 3.2.1. After entering the index you will be prompted to edit the first part of the equation X(t) (in terms of t). Press `return`

when done. Now you will be prompted to edit the second part of the equation Y(t) (in terms of t). Press [return] when done.

3.4.2 Overlay Parametric Pressing [O] will allow you to overlay a *parametric equation* in the same manner you did in section 3.2.8. Note that there are only four indices not eight as in all the other graphers.

3.4.3 Change t Range Select [E] to change the bounds of t. The default bounds on the parameter t are $-10 < t < 10$. After pressing [E], you will see the current range of t, and then you will be prompted to enter tmin and tmax. These parameters can be any real number or an algebraic expression like "pi*2".

3.5 POLAR GRAPHER

When you are ready to enter the polar grapher, select [4] from the Main Menu. There will be a slight delay while the polar grapher program is loaded. The initial screen (except for the polar equation graphed) will look like the one shown in figure 3.5.

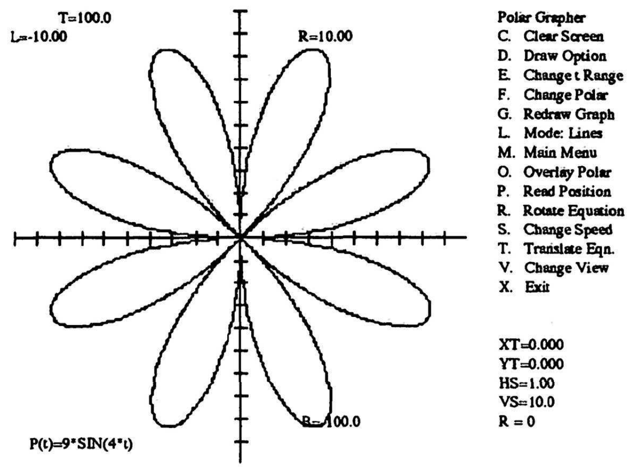


Figure 3.5

Notice the changes in the polar grapher menu versus the function grapher menu. The "A. Transform F(x)" and "I. Inverse F(x)" options have been removed. The "F. Change F(x)" is now "F. Change Polar" and the "O. Overlay F(x)" is now "O. Overlay Polar". Notice the "F(x)" in rotate and translate has been changed to equation. Finally, notice the new feature "E. Change t Range".
In the following sections we will only discuss these features as all the others have not changed. Note that rotate and translate have not been changed only the wording in the menu has been changed.

3.5.1 Change Polar Pressing [F] will allow you to change or display any of the eight *polar equations* in the same manner as you did in section 3.2.1.

3.5.2 Overlay Polar Pressing [O] will allow you to overlay a *polar equation* in the same manner as you did in section 3.2.8.

3.5.3 Change t Range Pressing [E] will allow you to change the t range in the same manner as you did in section 3.4.3.

3.6 SPECIAL KEYS

This section will discuss special keys available to the user.

Print Graphics Screen: If you have installed "graphics" you can use the [shift] [PrtSc] keys to get a screen dump.

Abort Plot: To STOP the plot at any time during the plotting routine and return to the menu, press the [Esc] key or [E] key.

Pause Plot: To interrupt (PAUSE) the plot, press the $\boxed{\text{spacebar}}$. Pressing the spacebar again or any other key will RESUME the plot.

3.7 3–D GRAPHER INTRODUCTION

A great deal is known about single variable function graphers. The use of such graphers and understanding about how they can enhance the teaching and learning of mathematics are on the rise. Much less is known about the use of surface graphers, that is, devices that produce a graph of a function of two variables. However, several things about graphing functions of two variables are very clear. Obtaining graphs by hand is a difficult task for both student and teacher. Students have a good bit of trouble visualizing in three dimensions. Teachers have a rough time producing quick, accurate graphs of functions of two variables.

The three-dimensional grapher described in this manual is designed to allow the user to obtain reasonably accurate graphs of functions of two variables. The user can obtain the graph for $a \leq x \leq b$, $c \leq y \leq d$ and $e \leq z \leq f$, and then choose an arbitrary point in three dimensional space from which to view the graph. Once the first graph is drawn, the points are stored in an array so that the graph can be redrawn quickly from different views. The user can choose any point in three-dimensional space from which to view the graph. The resolution of a graph is under user control.

This three-dimensional grapher allows the user to interactively explore the behavior of surfaces. Local maximum and minimum values of functions of two variables can be investigated graphically. The grapher can help students deepen understanding and intuition about functions of two variables. It can provide a geometric representation of problem situations to go along with an algebraic representation. The connections between these two representations can be explored and exploited to gain better understanding about problem situations.

The single most important feature of this graphing program is that virtually every aspect of this utility is interactive and under user control. This utility was designed to help teachers teach and students learn mathematics in an atmosphere where both are active partners in the educational experience.

We will now describe how the user chooses a region of three-dimensional space in which to draw a graph of a function of two variables, and the way in which that graph can be viewed.

Definition. The set $\{(x, y, z) \mid a \leq x \leq b,\ c \leq y \leq d,\ e \leq z \leq f\}$ is called the *viewing box* $[a, b]$ by $[c, d]$ by $[e, f]$.

Notice that the viewing box [a,b] by [c,d] by [e,f] is completely determined by the points $a \leq x \leq b$, $c \leq y \leq d$, $e \leq z \leq f$. The user can change the viewing box by selecting key $\boxed{\text{V}}$ and entering the values of $a, b, c, d, e,$ and f. The default viewing box is [-10, 10] by [-10, 10] by [-10, 10].

Next, the user decides how to view the graph contained in the selected viewing box. Two points can be selected. The point at which the user places his/her "eye" is called the *viewing point.* The point at which the view of the eye is directed is called the *aiming point.* The aiming point can be changed by selecting key $\boxed{\text{A}}$ and inputting the rectangular coordinates of the point. The viewing point can be changed by entering the spherical coordinates (d, ϕ, θ) of the point using keys $\boxed{\text{Z}}$, $\boxed{\text{R}}$, and $\boxed{\text{E}}$. The

"Change Elevation" key [E] allows the user to select the angle ϕ. ϕ is the angle the line through the origin and the viewing point makes with the z-axis (Figure 3.6). The "Change Rotation" key [R] allows the user to select the angle θ. θ is the angle between the x-axis and the plane perpendicular to the xy-plane which contains the viewing point (Figure 3.6). Positive direction is counterclockwise. Finally, the "Chg. Dist. (Zoom)" key [Z] allows the user to select the distance that the viewing point is from the aiming point. The default aiming point is (0, 0, 0) and the default viewing point is $(100, 60°, 30°)$.

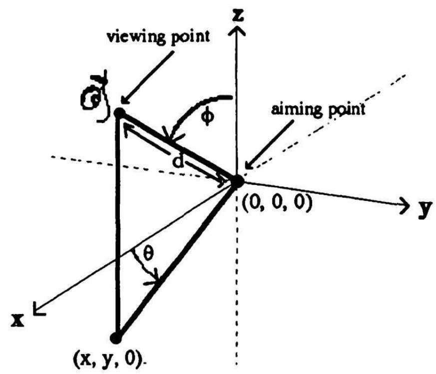


Figure 3.6

The software draws the graph, in a cone of vision determined by the aiming point and the viewing point. The size of the cone is an automatic feature and cannot be selected by the user. The viewing point is the vertex of the cone, and the line determined by the two points is the axis of the cone. The view is from the viewing point toward the aiming point. Selecting [H] allows the user to view the graph with or without hidden lines. That is, if the *hidden lines* option is on, then the user will *not* see the portions of the surface that should be hidden from view by other portions of the surface. Basically, the graph that the user sees is the intersection of the software-selected cone of vision with the user-selected viewing box. By changing the aiming point, viewing box, and viewing point, the user can view *any* portion of a surface with a high degree of resolution.

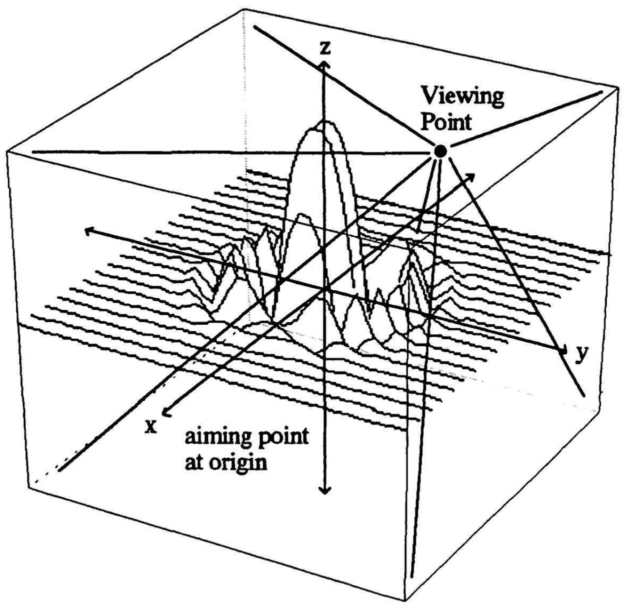


Figure 3.7

This drawing and viewing feature of the software literally allows the user to move around and view the surface as if in an airplane. You can move closer or farther away, and view the graph above or below by careful selection of elevation, rotation, and distance (Figure 3.7).

3.8 3–D GRAPHER START UP

Start Up: Exit the *Master Grapher* program by pressing [X] and then do the following. If you are using the $5\frac{1}{4}''$ disk, insert the *3-D Grapher* disk. Otherwise type either *3DG.EXE* or *3DG-H.EXE* if you have a Hercules graphics card. Once you start the program, information about the graphing program will appear on the screen. Press any key and a graph will be drawn.

3.9 3–D GRAPHER MENU

3.9.1 Aiming Point: Select [A] to choose a new point at which the viewing eye will be focused. You will need to enter three parameters the x, y, and z which represent the rectangular coordinates of the desired aiming point. The default aiming point is (0, 0, 0).

3.9.2 Clear Screen: Select [C] to clear the current graphing window.

3.9.3 Set Cuts (Draw): Select [D] to choose the axis through which the points will be plotted. Default is "x-cuts" where sections are taken parallel to the yz plane.

3.9.4 Elevate: Select [E] to change the elevation or the angle ϕ of the viewing point. ϕ is the angle the line through the origin and the viewing point makes with the z-axis. The default elevation angle ϕ is $60°$.

3.9.5 Change F(x,y): Press [F] to change the function to be graphed. You will see a menu with 10 options. Pressing [1] through [8] will select the function to be plotted. Only one *Selected* function can be plotted. Thus, you can *predefine* up to 8 functions and then plot them individually. Press [9] to return to the default menu.

Select [0] to change any one of the eight functions listed. When you press [0], you will then be asked which function index you wish to change. (There will be no prompt). You will now be able to edit the function you specified; to do this use the keys in the table below. Edit the equation you will want to graph (in terms of x and y) and press [return].

See section 3.2.1 for a list of the editing keys, the special symbols, and the built–in Functions.

3.9.6 Redraw Graph: Select [G] to replot the surface.

3.9.7 Hidden Lines: Select [H] to add or delete the hidden line subroutine. This command is basically a toggle switch between *Hidden Lines* and *No Hidden Lines.*

3.9.8 Add Axis: Select [I] to add the three coordinate axes to the graph.

3.9.9 Lines/Grid: Selecting [L] will bring up two menus, a "Line Menu" and a "Grid Menu", with six options under the line menu and three under the grid menu. The line menu will allow you to graph a line in 3–D space, where the user will supply the desired plane and the position on the plane. After this is done the line will be drawn from the range of the viewing cube plus four. For example, if you wanted to draw a line from (1,2,zmin-2) to (1,2,zmax+2) you would do the following: press [L] to enter the "Lines/Grid" menu, press [4] to draw a line "IN xz PLANE AT x", enter the x parameter by pressing [1] [return], enter the y parameter by pressing [2] [return]. The line will now be drawn given the parameters. The "Grid Menu" has three options. Simply select the plane you want a grid drawn in by selecting one of the three options. At this point you will be prompted for the plane position. Type any real number or algebraic expression and the grid will appear at that position in the desired plane.

3.9.10 Rotate: Select [R] to change the rotation angle θ. θ is the angle the plane perpendicular to the xy-axis that contains the viewing point makes with the x-axis. The default rotation angle θ is $30°$.

3.9.11 Resolution (Speed): Select [S] to change the resolution (that is, the number of sections and number of points per section). When you select [S] you will be asked to enter two parameters, the "Number of Sections", and the "Number of Points/Section". The number of sections refers to the number of x-sections (x-cuts or x-wires), that is, sections *parallel* to the yz plane. A given x-section, say $x = a$, is the graph of $z = f(a,y)$. (The intersection of the graph of the surface and the plane $x = a$). The larger

the number, the slower the plotting speed, but you get better resolution with more sections. The number of points/section refers to the number of points that are to be drawn in one x-section, or yz plane. Again, more points means slower plotting speed but better resolution.

3.9.12 Defaults: Select $\boxed{\text{T}}$ to return to the default mode for all features.

3.9.13 Change Viewing box: Select $\boxed{\text{V}}$ to change the viewing box. When you select $\boxed{\text{V}}$ you will see the old viewing box displayed and then you will be prompted to input *xmin*, *xmax*, *ymin*, *ymax*, *zmin*, *zmax* (in that order). Think of *xmin* as "back" and *ymin* as "left", etc. (See Figure 3.8). If you want the new viewing box to be [-20, 30] by [-15, 40] by [-10, 10] you will enter $\boxed{-}$ $\boxed{2}$ $\boxed{0}$ $\boxed{\texttt{return}}$ $\boxed{3}$ $\boxed{0}$ $\boxed{\texttt{return}}$ $\boxed{-}$ $\boxed{1}$ $\boxed{5}$ $\boxed{\texttt{return}}$ $\boxed{4}$ $\boxed{0}$ $\boxed{\texttt{return}}$ $\boxed{-}$ $\boxed{1}$ $\boxed{0}$ $\boxed{\texttt{return}}$ $\boxed{1}$ $\boxed{0}$ $\boxed{\texttt{return}}$. The new graph will immediately be calculated and then drawn.

3.9.14 Exit: Select $\boxed{\text{X}}$ to exit the program and return to the IBM^R Operating System.

3.9.15 Change Distance (Zoom). Select $\boxed{\text{Z}}$ to change the *distance* from the viewing point to the aiming point. A distance of 100 causes the viewing box [-10, 10] by [-10, 10] by [-10, 10] to approximately "fill" the computer screen. The default zoom distance is 100.

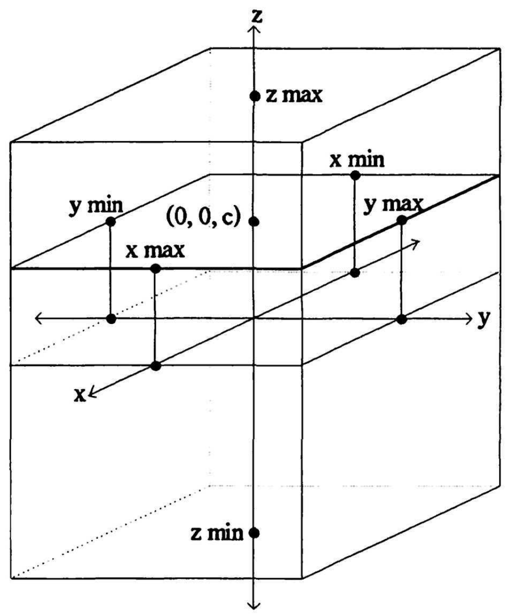


Figure 3.8

3.9.15 Change Distance (Zoom). Select $\boxed{\text{Z}}$ to change the *distance* from the viewing point to the aiming point. A distance of 100 causes the viewing box [-10, 10] by [-10, 10] by [-10, 10] to approximately "fill" the computer screen. The default zoom distance is 100.

3.10 SPECIAL FEATURES FOR THE 3–D GRAPHER

This section will discuss the special features available to the user.

3.10.1 SPECIAL KEYS

Abort Plot: To STOP the plot at any time during the plotting or computation of the graph, press the $\boxed{\text{Esc}}$ key or $\boxed{\text{E}}$ key.

Pause Plot: To interrupt (PAUSE) the plot, press the $\boxed{\texttt{spacebar}}$. Pressing the spacebar again or any other key will RESUME the plot. The x-, y-, and z-coordinates of the last point plotted (in the current section being plotted) will be displayed.

3.10.2 HELPFUL HINTS

1) Do not turn the hidden line option on (key $\boxed{\text{H}}$) until you finish your exploration because the plot will be slower with the hidden line option active.
2) Because of the increased plotting time, increase resolution only when you desire a "nice" plot.
3) To plot a *sphere*, for example, $z^2 + y^2 + x^2 = 70$ (and other similar quadratic surfaces), first plot $z = (70 - x^2 - y^2)^{(1/2)}$. Then select $\boxed{\text{F}}$ to change the "displayed" function to $z = -(70 - x^2 - y^2)^{(1/2)}$.

Finally, click `previous menu` and the "lower" hemisphere will be overlayed on the existing "upper" hemisphere.

4) Use the zoom-in key `Z` to change the cone of vision. Changing the distance d allows you to see more, or less, of the graph that is contained in the viewing box.

5) To zoom-in on features away from the center (aiming point) of a current graph, you may need to change both the viewing box *and* the aiming point. However, do this in low resolution until you are sure you have selected the viewing box and the aiming point that displays the desired features.

Note: AT&T users

If you are using an AT&T series computer with an 8087 math coprocessor you will need to disable the math coprocessor for the software to run. This can be accomplished by typing "set no87=suppressed`return`". To enable the 8087 after the program is finished, simply type "set no87=`return`".

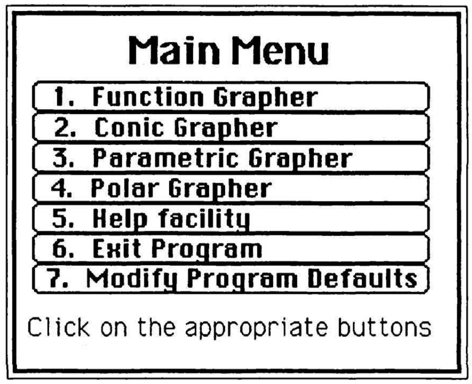
Chapter 4

Master Grapher Version 1.0 for the MACINTOSH^{TM}

4.1 MASTER GRAPHER START UP

IMPORTANT: Do NOT write protect either the *MASTER GRAPHER*, or *3-D GRAPHER* disk, the program writes to the disk!!!!

Start Up: Boot with a system disk (use system 4.1 and finder 5.5 or better), then insert the *Master Grapher* disk. If you are not using the recommended system or are using a Mac 512 you will need to **use the *Master Grapher* 512k disk (disk 2)**, otherwise **use the *Master Grapher* 800k disk (disk 1)**. To determine which system and finder you are using pull down the Apple menu and get About Finder while in the finder. A dialog box will appear with the finder number on it. If it is less than 5.5 use *Master Grapher 512*. Once you start the program information about the graphing program will appear on the screen. Click the mouse or press any key to go to the *main menu*.



3-D Grapher: *Master Grapher* includes *3-D Grapher*, a graphing utility for functions of *two* variables. The *3-D Grapher* utility is described in section 4.8 - 4.12 on pages 64-69.

4.2 MODIFYING THE PROGRAM DEFAULTS

Before running any of the graphing programs on this disk you may choose to modify some of the program defaults. When you press [7] or click the mouse on *7. Modify Program Defaults* you will see figure 4.1.

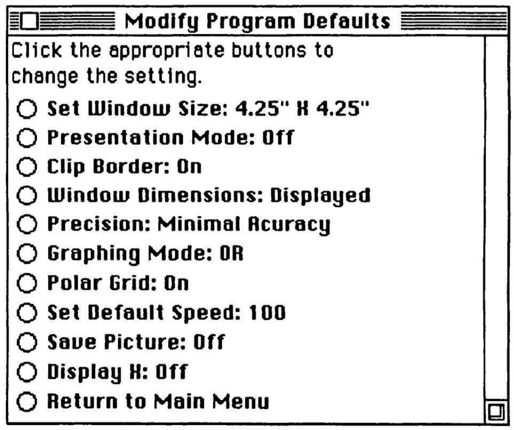


Figure 4.1

4.2.1 Set Window Size: The first option on the list, *Set Window Size*, can be used to set the size of the graphing window. When you click the *Set Window Size* button, the screen will clear and the current window size and the maximum window size will be displayed. The program will then prompt you to enter the width and then the height of the window (in terms of inches). There are advantages to a square screen, but you can choose any size window provided it fits within a minimum of 0.75 inches and a maximum dimension which is determined by the machine you are using. If you enter a dimension which is too small or too large the program will choose the maximum viewing size for that dimension.

4.2.2 Presentation Mode: The next option, *Presentation Mode* allows the user to make the grapher more visible if the user is displaying it on an overhead projector (in *On* position). This is achieved in the program by using thicker lines to draw the graph and using larger and boldface text. The thicker lines are not as clear on the screen, so this option should only be used for presentation purposes.

4.2.3 Clip Border: The next option, *Clip Border* is used to toggle between *Clip Border: On* and *Clip Border: Off.* When you use the option *Clip Graphics Screen*, the graph is copied to the clipboard and can be carried into the scrapbook. The clip border option lets you decide whether it will be copied with or without the border.

4.2.4 Window Dimensions: *Window Dimensions: Displayed* gives you the option of having the L, R, B, and T values displayed in the viewing rectangle or having the *Window Dimensions: Not Displayed* so that the screen is less cluttered. If these parameters are not displayed, the equation of the function being graphed is also not displayed. However, if either dimension of the window is less than or equal to 2.5 inches the dimensions are not displayed in the window but the function is displayed in another window. When you use *Master Grapher* to draw a graph to be used for demonstration, you may prefer to use *Window Dimensions: Not Displayed.* However, when you are using *Master Grapher* to explore and discover the behavior of a relation, particularly when zooming in or out, you will most likely choose to use *Window Dimensions: Displayed.*

4.2.5 Precision: When you use *Window Dimensions Not Displayed*, notice that the next option becomes shadowed. This option, *Display Minimal Accuracy*, is available only with *Window Dimensions Displayed*, because it determines the accuracy of the numbers used to represent the L, R, B, and T values. *Precision: Minimal Accuracy* toggles between its default setting and *Precision: Full Precision*, and determines whether the L, R, B, and T values are given with full machine precision accuracy or with significant digit accuracy.

4.2.6 Graphing Mode: The *Graphing Mode: OR* is a toggle between *Graphing Mode: OR* and *Graphing Mode: XOR* that determines whether the intersection of two lines is represented as an open pixel (XOR, exclusive or) or with merely a crossing of lines with no change in the pixels at the intersection (OR, or).

4.2.7 Polar Grid: Setting *Polar Grid: On* will cause a polar grid to be used in the polar grapher instead of the rectangular one that is used in all the other graphers. If *Polar Grid: Off* is set then a rectangular grid will be drawn in the polar grapher.

4.2.8 Default Speed: The *Set Default Speed* option will allow you to modify the program's default speed. This is not a direct change to the default speed but a rule change, where the following rule is applied.

Rule 1	- Function grapher:	Factor * 1 * Window Size/4.45
Rule 2	- Conic & Parametric grapher:	Factor * 0.5 * Window Size/4.45
Rule 3	- Polar grapher:	Factor * 2 * Window Size/4.45

Where Window Size is the width of the window. For example, to obtain a default speed of 100 in the conic grapher given the width of the window to be 4.45 inches, the default speed setting must be 200.

4.2.9 Save Picture: The *Save Picture* option saves the graphing window to disk when the user clicks on any of the buttons in the grapher menu. This option can be useful to instructors, but it slows the program down and can use quite a bit of disk space. It is recommended that you have a hard drive when you use this option because each file uses a minimum of 1 K of disk space, but on the average it uses 4K of disk space. The files are saved in Microsoft^{TM} Basic graphic file format so you can write a basic program to view them.

4.2.10 Display X: The next option, *Display X*, will allow the user to watch the values of x or t as the graph is being drawn. Although this is quite a useful item, it is recommended that you leave it off unless you have an accelerated Mac + or SE or you have a Mac II or better because of speed considerations.

4.2.11 Return to Main Menu: This last option will return you to the main menu.

4.3 FUNCTION GRAPHER

When you are ready to enter the Function Grapher, select $\boxed{1}$ from the Main Menu. The initial screen (except for the function graphed and this viewing rectangle) will look like the one shown in the figure 4.2.

The available interactive commands will be in the window on the right side of the screen. In the bottom of this window are listed the distance between horizontal scale marks (HS) and the distance between the vertical scale marks (VS), as well as the translation factors (XT and YT), and the rotation factors (R).

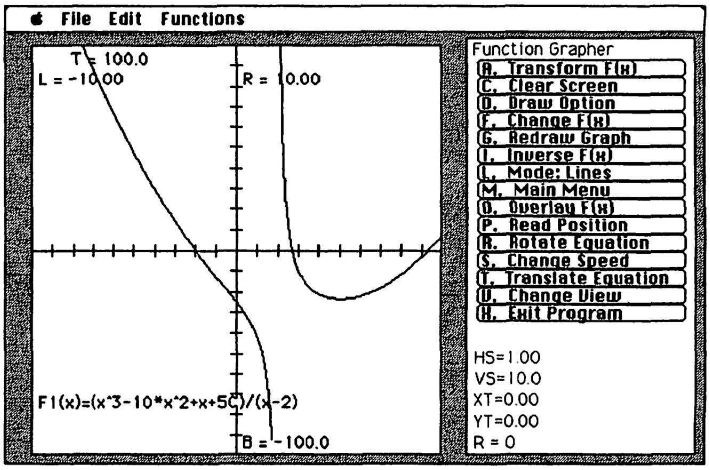


Figure 4.2

4.3.1 Change F(x): Select $\boxed{F}$ to change the function to be graphed. You will see a menu like the one in Figure 4.3. Clicking on the buttons **F1(x)=** through **F8(x)=** will either select or deselect them. Only the *Selected* function(s) will be plotted. Thus, you can *predefine* and plot up to 8 functions at the same time. Clicking on the *Previous Menu* button returns you to the default menu.

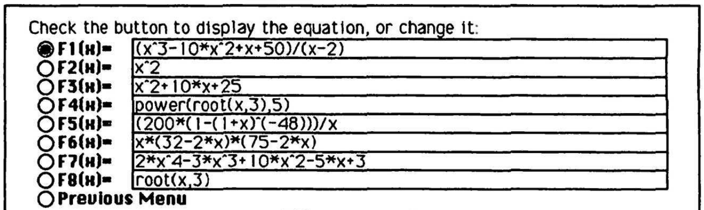


Figure 4.3

To edit an equation, click the mouse on the boxed equation you want to change. This is called an edit field. You will now see either an area of the equation highlighted or a text cursor. To change the whole equation simply use the mouse to highlight all of the text and then start typing the new equation. The *Cut, Copy, Paste* option under the edit menu will function properly while editing the equation. Make note the $\boxed{\texttt{tab}}$ does not move you from field to field, and the $\boxed{\texttt{return}}$ does not enter the equation, like it does elsewhere in the program.

Use standard BASIC syntax to enter the desired equation, for example $x^4 - 3x^2 + 15$ would be entered by pressing $\boxed{x}$ $\boxed{\wedge}$ $\boxed{4}$ $\boxed{-}$ $\boxed{3}$ $\boxed{*}$ $\boxed{x}$ $\boxed{\wedge}$ $\boxed{2}$ $\boxed{+}$ $\boxed{1}$ $\boxed{5}$. The constants e and π are entered as $\boxed{e}$ and $\boxed{p}$ $\boxed{i}$, respectively. Special keying instructions are needed to enter built–in functions or to obtain correct graphs of some special functions. Here is a list of the special functions.

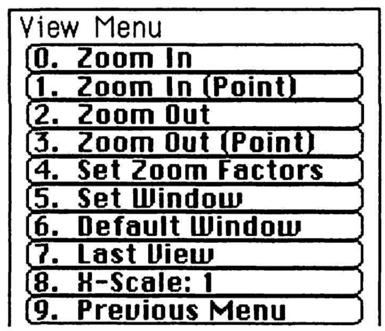
Special Symbols & Built–in Functions:

1. $+$ is addition.
2. $-$ is subtraction.
3. $*$ is multiplication.
4. $/$ is division.
5. $\wedge$ is x^a.
6. $\backslash$ is Integer Division.
7. ABS($x-2$) is $|\,x-2\,|$.
8. CEIL(x) = $[\![x+1]\!]$.
9. EXP(x) is e^x.
10. FIX(x) is FLOOR if $x > 0$, and CEIL if $x < 0$.
11. FLOOR(x) = $[\![x]\!]$.
12. INT(x) is the greatest integer function .
13. LOG(x) is $\ln x$.
14. LOG10(x) is $\log_{10}(x)$.

15. LOG2(x) is $\log_2(x)$.
16. ROUND(x) rounds to the nearest integer.
17. SGN(x) is the signum function.
18. SQR($x+6$) is $\sqrt{x+6}$.
19. SIN(x) is sin x . All other trigonometric functions are entered in the same manner (e.g. arctan x is ARCTAN(x), cosh x is COSH(x), etc.). Here is a list of all the trigonometric functions supported: arccos, arccosh, arccot, arccoth, arccsc, arccsch, arcsec, arcsech, arcsin, arcsinh, arctan, arctanh, cos, cosh, cot, coth, csc, csch, sec, sech, sin, sinh, tan, tanh.

Special Functions:

20. LOGB(x, a) is $\log_a(x)$.
21. ROOT(x, a) is $x^{1/a}$. Note: you can enter $x^{1/a}$ as $x \wedge (1/a)$, but you will only get the portion of the graph in the first quadrant.
22. POWER(x, a) is x^a .



4.3.2 Change View: Selecting [V] will display the menu to the right. You may have changed the speed, the plot mode or a variety of different commands that do not cause the graph to be immediately replotted. The function grapher incorporates those changes when the graph is redrawn or the viewing rectangle is changed. Suppose we want to view the graph of

$$f(x) = \frac{x^3 - 10x^2 + x + 50}{x - 2}$$

in the viewing rectangle $[-1, 1]$ by $[-1, 1]$. To do this enter f by selecting [F] and proceeding as detailed in the "Change F(x)" Section 4.3.1 (page 57), then select [V]. You can now choose the method you want to change the viewing rectangle.

0. Zoom-In: To set your own area for the new viewing rectangle, select [0]. Move the mouse to the desired upper left hand corner of the new viewing rectangle within the current view. Now press the mouse button and, while holding it down, drag the mouse to the lower right hand corner of the viewing rectangle. When you release the mouse button you will now see the function plotted in the new viewing rectangle.

An illustration of the effect that "zoom-in" can have on a given view appears in Figure 4.4. The function $f(x) = x^2 - 3x + 2$ was graphed in the standard viewing rectangle, then graphed again in the $[0.8, 2.3]$ by $[-1, 0.5]$ viewing rectangle. The behavior of the graph between $x = 1$ and $x = 2$ is much more obvious in the zoom-in view.

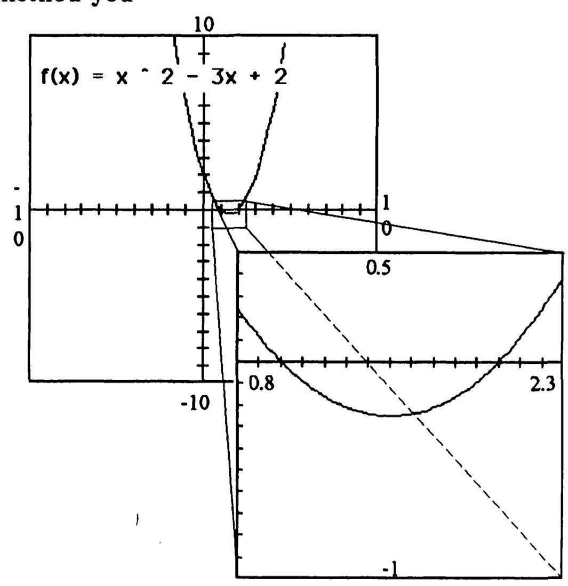


Figure 4.4

1. Zoom In (Point): This command is useful when you want a close-up or magnified view of a particular section of a graph around a particular point. Select [1]. Now move the mouse to the point you want to be the center of the new viewing rectangle

and click the mouse. The function will be replotted in the new viewing rectangle with the point you selected in its center. The *size* of the viewing rectangle will be determined by the zoom factor settings [4].

2. Zoom Out: When you select [2], the graph will be immediately redrawn in a viewing rectangle expanded by the zoom factors you set with key [4] (or the default values of 10 for x and 10 for y).

3. Zoom Out (Point): When you select [3], you will be asked to click the mouse in the current viewing rectangle at the location about which you wish the "zoom-out" to be centered. Once again the zoom factor will be determined by the default factors (both 10) or the factors you set using [4].

4. Set Zoom Factors: This command sets the horizontal (x) and vertical (y) zoom factors. When you select [4] you will see the old settings for x and y zoom factors and then you will be prompted to enter new ones. This can be done by typing in any real number or algebraic expression. Assume the current viewing rectangle is [L, R] by [B, T]. Enter the value you wish L - R and B - T to be multiplied by. Values greater than 1 cause the horizontal or vertical size of the rectangle to increase (zoom-out), and values less than 1 cause the horizontal or vertical size of the rectangle to decrease (zoom-in). The change in size of the viewing rectangle is symmetric about the center of the rectangle. Sometimes you will want the zoom factors on the x-axis and y-axis to be different. Note: you can "zoom-in" using "zoom-out" by selecting zoom factors less than one.

5. Set Window: To change the viewing rectangle, select [5]. The menu will clear and an *OK* button and four edit fields labelled *L, R, T, B* will appear. Change the values using the edit menu, the mouse, and the keyboard. You can move to a different field by either pressing [tab], or click on the other field. When you are done editing the values simply press [return], or click the *OK* button. The values can be any real number or algebraic expression. The screen will be redrawn in the specified viewing rectangle and the "View Menu" will return.

6. Default Window: When you select [6] the displayed function(s) will be replotted in the default viewing rectangle $[-10, 10]$ by $[-10, 10]$, speed, mode: lines, rotation $0°$, and translation $(1,1)$. After the screen is redrawn you will be returned to the "View Menu".

7. Last Window: When you select [7] the displayed function(s) will be replotted in the last viewing rectangle. However, speed, mode, rotation, and translation will remain unaltered from the current settings. After the screen is redrawn you will be returned to the "View Menu".

8. X-Scale: When you select [8], the distance between the tick marks on the x-axis will be changed. If X-Scale is in units of $\frac{\pi}{2}$ the distance between tick marks will be in a factor of units of either $\frac{\pi}{2}$ or $\frac{\pi}{4}$. If X-Scale is in units of 1 the distance between tick marks will be in a factor of units of either 1 or 0.5. The graph will be redrawn immediately with the appropriate x-scale. This option is available only in the function grapher.

9. Previous Menu: When you select [9], you will return to the grapher menu. This option is [8] on all the other graphers.

4.3.3 Change Speed: To change the plotting speed, select [S]. You will now see the current plotting speed, the minimum and maximum allowable plotting speeds, and a prompt to enter a new speed. Enter the speed you want by typing either a number or an algebraic expression with terms pi, e, or some constants and then press [return]. The program will evaluate the expression and enter the resulting value as the new plotting speed. (You may wish to experiment with various settings until you find the one you prefer.) The default speed is a good compromise between speed and resolution.

4.3.4 Mode: Lines or Points: Select L to choose one of two plotting modes. One plots only the points evaluated for a particular function, whereas the second connects each consecutive pair of points with a line segment (the default mode).

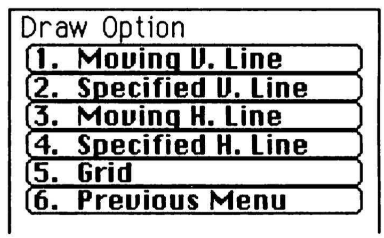
4.3.5 Rotate F(x): Select R to rotate the function about the origin. When you select R, you will need to enter the counterclockwise rotation angle in degrees. This can be either a real number or an algebraic expression like "pi*2". A positive input will produce a counterclockwise rotation and a negative input will be clockwise.

4.3.6 Translate F(x): Select T to draw a graph of the function $y = f(x)$ translated H units horizontally and V units vertically. After selecting T, enter the amount of horizontal translation and press return. Then enter the amount of vertical translation and press return. Nothing will appear to happen. To see the effect of the translation, you will need to redraw the graph G. You will now have graphed the *displayed* function with the translation factors applied. If you wish to view both the original function and the translated function, select O (Overlay F(x)) and choose the index of the original function and then O W/O Rot. & Trans. In this manner you can get both the function and its translation in the same viewing rectangle.

4.3.7 Redraw Graph: Redraws the graph with the current settings.

4.3.8 Overlay F(x): When you select O you will need to select an index function to overlay and then press return. For example, if you choose 6, then the sixth function in the function menu will be plotted on the same screen with the function(s) already plotted. Once the index is selected you will need to tell the program whether you want the function rotated and translated or not. If there are non-zero values in the XT, YT, and R categories in the lower right hand corner of the display screen, then choosing 1 and pressing return will plot the selected function *with* those rotations and translations applied. Choosing 0 will have the function plotted *without* any rotation or translation applied.

4.3.9 Clear Screen: Select C to clear the graphing window.



4.3.10 Draw Option: When you select D, a menu like the one to the right will appear with a list of options. Use 1 through 4 to approximate the coordinates of a point on the graph. To approximate the y-coordinate, a horizontal line can be specified to move up and down (use the ↑ or U to move the line up, the ↓ or D to move the line down, and S to change the rate of movement of the line) until it is fixed in position by pressing any other key. The y-coordinate of each point on the horizontal line will be displayed. The x-coordinate can be found in a like manner using a vertical line (use the → or R and the ← or L to move it right and left). Select 5 to overlay a lattice (an array of dots) in the viewing rectangle. The *distance* between these dots will be given by the HS (horizontal scale) and VS (vertical scale) values found in the lower right-hand corner of the screen. This command will help estimate the coordinates of a point on a graph. For example, zoom-in on some area of the current graph that is of interest to you. Select D 5 to obtain a lattice in the viewing rectangle. Use the HS and VS values to read the coordinates of the point you selected.

4.3.11 Read Position: The next option, P, can also be used to approximate the coordinates of any point in the current viewing rectangle. After selecting P simply click the mouse on the desired position in the graphing window. In the lower right-hand corner you will now see the location of the cursor in the viewing rectangle. To read another position, simply follow the procedure again.

4.3.12 Inverse F(x): If you wish to view the inverse relation (y, x) where $y = f(x)$ of any function in the function menu, select $\boxed{\text{I}}$. Then select the index of the function you wish to invert and press $\boxed{\texttt{return}}$, and the inverse *relation* will be overlayed immediately.

4.3.13 Transform F(x): This command allows you to draw the graph of $y = \mathrm{A}\, f\,(\mathrm{B}\,x\; + \mathrm{C}) + \mathrm{D}$ by specifying f, A, B, C, and D. Press $\boxed{\text{A}}$ or click the mouse on *A. Transform F(x).* You will then be asked to enter the index of the function f you wish to transform. For example, if you choose 6, then the sixth function in the function menu will be the function you are going to transform. Next, four edit fields and an *OK* button will appear. Enter the parameters A, B, C, and D. You may enter any real number or algebraic expression for example "pi*2". To move to a different field either press the $\boxed{\texttt{tab}}$ key or click the mouse on the other edit fields you want to modify. When all the parameters have been changed to the desired values press the $\boxed{\texttt{return}}$ key or click the *OK* button. The function $y = \mathrm{A}\, f\,(\mathrm{B}\,x\; + \mathrm{C}) + \mathrm{D}$ will be plotted immediately.

4.3.14 Main Menu: Select $\boxed{\text{M}}$ to return to the main menu.

4.3.15 Exit: Select $\boxed{\text{X}}$ will exit the program and return you to the system.

4.4 CONIC GRAPHER

When you are ready to enter the Conic Grapher, select $\boxed{2}$ from the Main Menu. There will be a slight delay while the conic grapher program is loaded. The initial screen (except for the conic equation graphed) will look like the one shown in the figure 4.5.

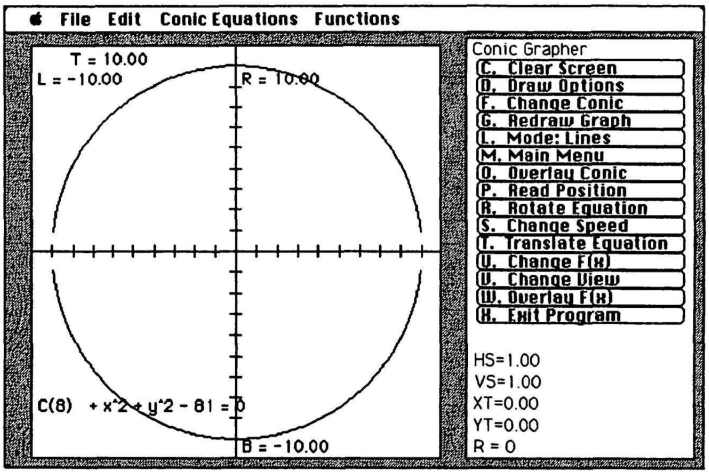


Figure 4.5

Notice the changes in the conic grapher menu versus the function grapher menu. The "A. Transform F(x)" and "I. Inverse F(x)" options have been removed. The "F. Change F(x)" is now "F. Change Conic" and the "O. Overlay F(x)" is now "O. Overlay Conic". Notice the "F(x)" in rotate and translate has been changed to equation. Two new options have been added: "U. Change Function(s)" and "W. Overlay Function(s)". In the following sections we will only discuss these features as all the others have not changed. Note: rotate and translate have not been changed, only the wording in the menu has been changed.

4.4.1 Change Conic: Pressing $\boxed{\text{F}}$ will clear the viewing rectangle and you will see a list of eight *conic equations.* Clicking on any of the C(1) - C(8) buttons will allow you to toggle between *Displayed* and *Not Displayed,* where an equation is displayed if it is selected. Clicking on *Previous Menu* will take you back to the conic grapher menu. When you click on *Change Conic Equation* you will see seven edit fields. The index of the function is the most important field. It must be set first because it determines the values of all the other parameters. The next six fields are the parameters A, B, C, D, E, and F. These parameters are the constants in the following equation: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. The constants may be entered

as real numbers or as algebraic expressions. Once the parameters are set, either press [return] or click the *OK* button. You will then see the changes appear in the menu.

4.4.2 Overlay Conic: Selecting [O] will allow you to overlay a *conic equation* in the same manner as you did with functions in section 4.3.8.

4.4.3 Change Function(s): Selecting [U] will allow you to change or display any of the eight *functions* in the same manner as you did in section 4.3.1.

Check the button to display the equation, or check change:
- ○ **Change Conic Equation**
- ● **C(1) + x^2 + y^2 - 81 = 0**
- ○ **C(2) - x + 3y - 5 = 0**
- ○ **C(3) + 2x^2 - y^2 + 3x - 5 = 0**
- ○ **C(4) - x^2 + 2y^2 + 3y - 5 = 0**
- ○ **C(5) + xy - x + 3y - 5 = 0**
- ○ **C(6) + x^2 + 2xy + y^2 + 2x + 2y - 81 = 0**
- ○ **C(7) + x^2 + 3xy + x + y = 0**
- ○ **C(8) - 2x^2 + 3y - 5 = 0**
- ○ **Previous Menu**

Figure 4.6

4.4.4 Overlay Function: Selecting [W] will allow you to overlay one of the eight *functions* in the same manner as you did in section 4.3.8.

4.5 PARAMETRIC GRAPHER

When you are ready to enter the parametric grapher, select [3] from the Main Menu. There will be a slight delay while the parametric grapher program is loaded. The initial screen (except for the parametric equation graphed) will look like the one shown in the figure 4.7.

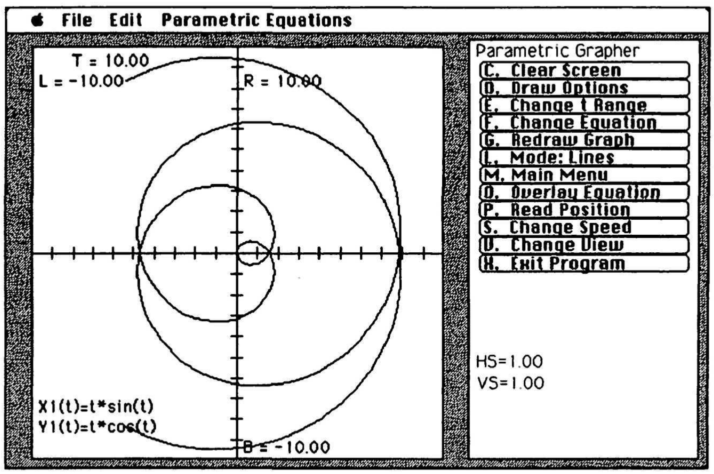


Figure 4.7

Notice the changes in the parametric grapher menu versus the function grapher menu. The "Transform F(x)", "Inverse F(x)", "Rotate Equation" , and "Translate Eqn." options have been removed. The "F. Change F(x)" is now "F. Change Parametric" and the "O. Overlay F(x)" is now "O. Overlay Parametric". One new option has been added: "E. Change t Range". In the following sections we will only discuss these new features as all the others have not changed.

4.5.1 Change Parametric: Select [F] to change the parametric equation to be graphed. You will see a menu with five buttons and eight edit fields. Selecting and deselecting [1] through [4] will determine which equation sets are to be graphed, the same way as you did in section 4.3.1. Thus, you can *predefine* and plot up to 4 parametric equations in this manner. Select [5] to return to the default menu.

To change any one of the four parametric equations listed, simply edit them in the same manner as you did in section 4.3.1. Remember that these are parametric equations so you need to edit the equation pair and use t as the variable.

4.5.2 Overlay Parametric: Selecting [O] will allow you to overlay a *parametric equation* in the same manner as you did in section 4.3.8. Note: there are only four indices, not eight as in all the other graphers.

4.5.3 Change t Range: Select [E] to change the bounds of t. The default bounds on the parameter t are $-10 < t < 10$. After selecting [E] you will see the current range of t and then you will be prompted to enter tmin and tmax. These parameters can be any real numbers or algebraic expressions like "pi*2".

4.6 POLAR GRAPHER

When you are ready to enter the polar grapher, select [4] from the Main Menu. There will be a slight delay while the conic grapher program is loaded. The initial screen (except for the polar equation graphed) will look like the one shown in the figure 4.8.

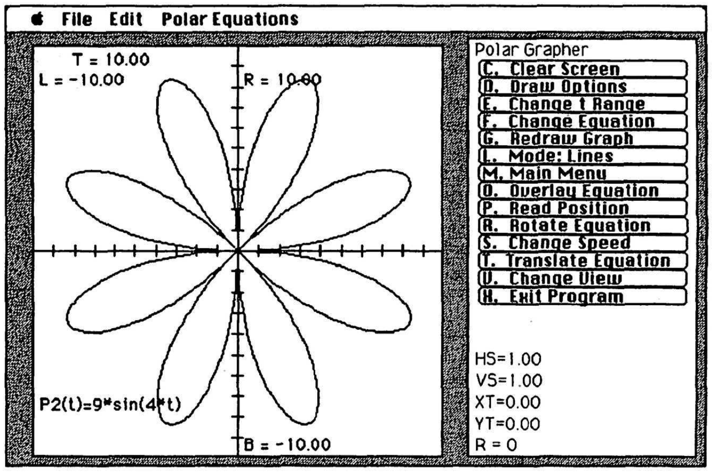


Figure 4.8

Notice the changes in the polar grapher menu versus the function grapher menu. The "A. Transform F(x)" and "I. Inverse F(x)" options have been removed. The "F. Change F(x)" is now "F. Change polar" and the "O. Overlay F(x)" is now "O. Overlay Polar". Notice the "F(x)" in rotate and translate has been changed to "Equation". Finally, notice the new feature "E. Change t Range". In the following sections we will only discuss these features as all the others have not changed. Note that rotate and translate have not been changed, only the wording in the menu has been changed.

4.6.1 Change Polar: Selecting [F] will allow you to change or display any of the eight *polar equations* in the same manner you did in section 4.3.1. Remember to use t as the variable, not x.

4.6.2 Overlay Polar: Selecting [O] will allow you to overlay a *polar equation* in the same manner you did in section 4.3.8.

4.6.3 Change t Range: Selecting [E] will allow you to change the t range in the same manner you did in section 4.5.3.

4.7 SPECIAL FEATURES FOR THE MASTER GRAPHER

This section will discuss the special features available to the user such as the menus and special keys.

4.7.1 File Menu

Quit: This option allows you to quit the program from almost anywhere. The exception to this rule is if you are entering a number in the program, you will be able to quit only after the number is entered.

Print Graphics Screen: This item will print the graphics window with or without a border (see section 4.2.3) to the available printer. The available printer is set by chooser.

Clip Graphics Screen: This item will clip the graphics window with or without a border. (See section 4.2.3 to set the clip border.) To use this option, select this menu item and then open up the desk accessory "Scrapbook". Paste the clipboard into the Scrapbook and then close the Scrapbook. It

is important to remember to close the Scrapbook. If it is left open, this program cannot clip to the clipboard. (In some paint programs like "SuperPaint^{TM}" this is also true.)

4.7.2 Edit Menu

Undo: This option is not available.

Cut: This allows you to cut the text out of any edit field.

Copy: This allows you to copy the text from an edit field to the clipboard.

Paste: This allows you to paste any text in the clipboard to an edit field.

4.7.3 Function Menu, Conic Equations, Parametric Equations, Polar Equations

All of these menus do basically the same thing: they allow you to select (or deselect) the equations that are going to be graphed when you select [G] *Redraw Graph.*

4.7.4 Special Keys

Abort Plot: To STOP the plot at any time during the plotting routine and return to the menu, press the [Esc] key or [E] key.

Pause Plot: To interrupt (PAUSE) the plot, press the [spacebar]. Pressing the spacebar again or any other key will RESUME the plot.

4.8 3–D GRAPHER INTRODUCTION

A great deal is known about single variable function graphers. The use of such graphers and understanding about how they can enhance the teaching and learning of mathematics are on the rise. Much less is known about the use of surface graphers, that is, devices that produce a graph of a function of two variables. However, several things about graphing functions of two variables are very clear. Obtaining graphs by hand is a difficult task for both student and teacher. Students have a good bit of trouble visualizing in three dimensions. Teachers have a rough time producing quick, accurate graphs of functions of two variables.

The three-dimensional grapher described in this manual is designed to allow the user to obtain reasonably accurate graphs of functions of two variables. The user can obtain the graph for $a \le x \le b$, $c \le y \le d$ and $e \le z \le f$, and then choose an arbitrary point in three dimensional space from which to view the graph. Once the first graph is drawn, the points are stored in an array so that the graph can be redrawn quickly from different views. The user can choose any point in three-dimensional space from which to view the graph. The resolution of a graph is under user control.

This three-dimensional grapher allows the user to interactively explore the behavior of surfaces. Local maximum and minimum values of functions of two variables can be investigated graphically. The grapher can help students deepen understanding and intuition about functions of two variables. It can provide a geometric representation of problem situations to go along with an algebraic representation. The connections between these two representations can be explored and exploited to gain better understanding about problem situations.

The single most important feature of this graphing program is that virtually every aspect of this utility is interactive and under user control. This utility was designed to help teachers teach and students learn mathematics in an atmosphere where both are active partners in the educational experience.

We will now describe how the user chooses a region of three-dimensional space in which to draw a graph of a function of two variables, and the way in which that graph can be viewed.

Definition. The set $\{(x,y,z) \mid a \le x \le b\ ,\ c \le y \le d\ ,\ e \le z \le f\}$ is called the *viewing box* $[a,b]$ by $[c,d]$ by $[e,f]$.

Notice that the viewing box [a,b] by [c,d] by [e,f] is completely determined by the points $a \le x \le b$, $c \le y \le d$, $e \le z \le f$. The user can change the viewing box by selecting key $\boxed{V}$ and entering the values of $a,b,c,d,e,$ and f. The default viewing box is [-10, 10] by [-10, 10] by [-10, 10].

Next, the user decides how to view the graph contained in the selected viewing box. Two points can be selected. The point at which the user places his/her "eye" is called the *viewing point.* The point at which the view of the eye is directed is called the *aiming point.* The aiming point can be changed by selecting key $\boxed{A}$ and inputting the rectangular coordinates of the point. The viewing point can be changed by entering the spherical coordinates (d,ϕ,θ) of the point using keys $\boxed{Z}$, $\boxed{R}$, and $\boxed{E}$. The "Change Elevation" key $\boxed{E}$ allows the user to select the angle ϕ. ϕ is the angle the line through the origin and the viewing point makes with the z-axis (Figure 4.9). The "Change Rotation" key $\boxed{R}$ allows the user to select the angle θ. θ is the angle between the x-axis and the plane perpendicular to the xy-plane which contains the viewing point (Figure 4.9). Positive direction is counterclockwise. Finally, the "Chg. Dist. (Zoom)" key $\boxed{Z}$ allows the user to select the distance that the viewing point is from the aiming point. The default aiming point is (0, 0, 0) and the default viewing point is $(100, 60°, 30°)$.

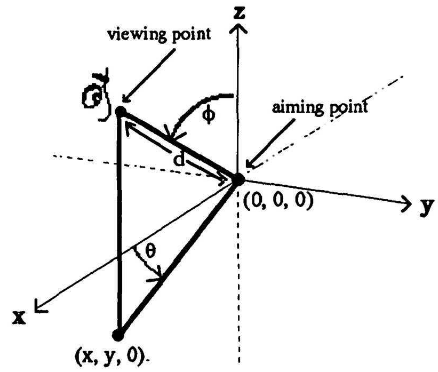


Figure 4.9

The software draws the graph, in a cone of vision determined by the aiming point and the viewing point. The size of the cone is an automatic feature and cannot be selected by the user. The viewing point is the vertex of the cone, and the line determined by the two points is the axis of the cone. The view is from the viewing point toward the aiming point. Selecting $\boxed{H}$ allows the user to view the graph with or without hidden lines. That is, if the *hidden lines* option is on, then the user will *not* see the portions of the surface that should be hidden from view by other portions of the surface. Basically, the graph that the user sees is the intersection of the software-selected cone of vision with the user-selected viewing box. By changing the aiming point, viewing box, and viewing point, the user can view *any* portion of a surface with a high degree of resolution.

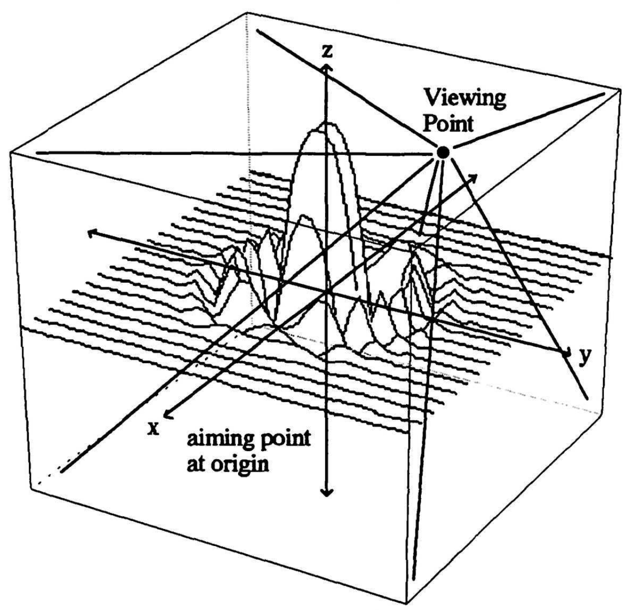


Figure 4.10

This drawing and viewing feature of the software literally allows the user to move around and view the surface as if in an airplane. You can move closer or farther away, and view the graph above or below by careful selection of elevation, rotation, and distance (Figure 4.10).

4.9 STARTING UP

Startup: Exit the Master Grapher program by pressing $\boxed{\text{X}}$ and then do the following. If you used *Master Grapher 512* then double click on the *3-D Grapher 512* icon, otherwise double click on the *3D Grapher* icon. To determine which system and finder you are using pull down the Apple menu and get About Finder while in the finder. A dialog box will appear with the finder number on it. If it is less than 5.5 use *Master Grapher 512.* Once the program starts, information about the graphing program will appear on the screen. Click the mouse or press any key and a graph will be drawn.

4.10 3–D GRAPHER MENU

4.10.1 Aiming Point: Select $\boxed{\text{A}}$ to choose a new point at which the viewing eye will be focused. You will need to enter three parameters the x , y , and z which represent the rectangular coordinates of the desired aiming point. The default aiming point is (0, 0, 0).

4.10.2 Clear Screen: Select $\boxed{\text{C}}$ to clear the current graphing window.

4.10.3 Set Cuts (Draw): Select $\boxed{\text{D}}$ to choose the axis through which the points will be plotted. Default is "x-cuts" where sections are taken parallel to the yz plane.

4.10.4 Elevate: Select $\boxed{\text{E}}$ to change the elevation or the angle ϕ of the viewing point. ϕ is the angle that the line through the origin and the viewing point makes with the z-axis. The default elevation angle ϕ is $60°$.

4.10.5 Change F(x,y): Select $\boxed{\text{F}}$ to change the function to be graphed. You will see a menu like the one in Figure 4.11. Clicking on the buttons **F1(x,y)=** through **F8(x,y)=** will select the function to be plotted. Only one *Selected* function can be plotted. Thus, you can *predefine* up to 8 functions and then plot them individually. Clicking on the **Previous Menu** button will return you to the 3–D menu.

Check the button to display the equation, or change it:
- ◉ **F1(x,y)=** 15*EXP(-0.04*(x^2+y^2))*cos(0.15*(x^2+y^2))
- ○ **F2(x,y)=** sqr(64-x^2-y^2)
- ○ **F3(x,y)=** (-sqr(64-x^2-y^2))
- ○ **F4(x,y)=** 0.25*x^2-y^2
- ○ **F5(x,y)=** x^2+y^2-9
- ○ **F6(x,y)=** x^2+y^2
- ○ **F7(x,y)=** 4*x^2-x*y+Y^2-x^3
- ○ **F8(x,y)=** 4*x^2-2*x*y-5*Y^2-3*x^3+2*y^3
- ○ **Previous Menu**

Figure 4.11

To edit in this menu, click the mouse on the boxed equation you want to edit. You will now see either an area of the equation highlighted or a text cursor. To change the whole equation simply use the mouse to highlight all of the text and then start typing the new equation. The *Cut, Copy, Paste* option under the edit menu will function properly while editing the equation. Note that the $\boxed{\texttt{tab}}$ does not move you from field to field, and the $\boxed{\texttt{return}}$ does not enter the equation, like it does in other places in the program.

See section 4.3.1 for the special symbols, and the built–in Functions.

4.10.6 Redraw Graph: Select [G] to replot the surface.

4.10.7 Hidden Lines: Select [H] to add or delete the hidden line subroutine. This command is basically a toggle switch between *Hidden Lines* and *No Hidden Lines.*

4.10.8 Add Axis: Select [I] to add the three coordinate axes to the graph.

4.10.9 Rotate: Select [R] to change the rotation angle θ. θ is the angle the plane perpendicular to the xy-axis that contains the viewing point makes with the x-axis. The default rotation angle θ is 30°.

4.10.10 Resolution (Speed): Select [S] to change the resolution (that is, the number of sections and number of points per section). When you select [S] you will be asked to enter two parameters, the "Number of Sections", and the "Number of Points/Section". The number of sections refers to the number of x-sections (x-cuts or x-wires), that is, sections *parallel* to the yz plane. A given x-section, say $x = a$, is the graph of $z = f(a, y)$, the intersection of the graph of the surface and the plane $x = a$. The larger the number the slower the plotting speed, but you get better resolution with more sections. The number of points/section refers to the number of points that are to be drawn in one x-section, or yz plane. Again, more points means slower plotting speed but better resolution.

4.10.11 Defaults: Select [T] to return to the default mode for all features.

4.10.12 Change Viewing box: Select [V] to change the viewing box. When you select [V] you will see the old viewing box displayed and then you will be prompted to input *xmin*, *xmax*, *ymin*, *ymax*, *zmin*, *zmax* (in that order). Think of *xmin* as "back" and *ymin* as "left", etc. (See Figure 4.12). If you want the new viewing box to be [-20, 30] by [-15, 40] by [-10, 10] you will enter [-] [2] [0] [return] [3] [0] [return] [-] [1] [5] [return] [4] [0] [return] [-] [1] [0] [return] [1] [0] [return]. The new graph will immediately be calculated and then drawn.

4.10.13 Exit: Select [X] to exit the program and return to the MacIntosh^{TM} Operating System.

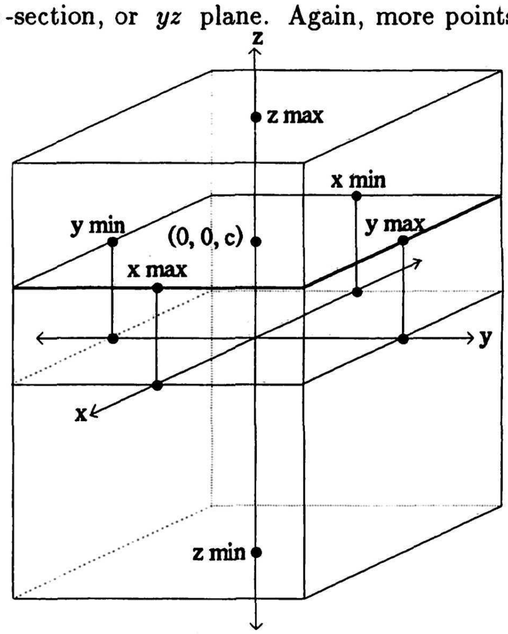


Figure 4.12

4.10.14 Change Distance (Zoom): Select [Z] to change the *distance* from the viewing point to the aiming point. A distance of 100 causes the viewing box [-10, 10] by [-10, 10] by [-10, 10] to approximately "fill" the computer screen. The default zoom distance is 100.

4.11 SPECIAL FEATURES FOR THE 3–D GRAPHER

This section will discuss the special features available to the user such as the menus and special keys.

4.11.1 File Menu

Quit: This option allows you to quit the program from almost anywhere. The exception to this rule is if you are entering a number in the program, you will be able to quit only after the number is entered.

Print Graphics Screen: This item will print the graphics window with or without a border to the available printer. The available printer is set by chooser.

Clip Graphics Screen: This item will clip the graphics window with or without a border. Here is how you should use this option. Select this menu item then open up the desk accessory "Scrapbook" now paste the clipboard into the scrapbook and then close the scrapbook. It is important to remember to close the scrapbook as if it is open this program can not clip to the clipboard. In some paint programs like "SuperPaint^{TM}" this is also true.

4.11.2 Edit Menu

Undo: This option is not available.

Cut: Allows you to cut the text out of any edit field.

Copy: Allows you to copy the text from an edit field to the clipboard.

Paste: Allows you to paste any text in the clipboard to an edit field.

4.11.3 Function Menu

This menu allows you to select and graph any of the eight equations with the current parameters in memory (rotation, elevation, resolution, etc.).

4.11.4 Line Menu

Draw line in XY plane parallel to X: Draws a line in the XY plane. The user will select the X value where the line will be drawn, and then select the Z parameter to determine which XY plane it is on.

Draw line in XY plane parallel to Y: Draws a line in the XY plane. The user will select the Y value where the line will be drawn, and then select the Z parameter to determine which XY plane it is on.

Draw line in XZ plane parallel to X: Draws a line in the XZ plane. The user will select the Z value where the line will be drawn, and then select the Y parameter to determine which XZ plane it is on.

Draw line in XZ plane parallel to Z: Draws a line in the XZ plane. The user will select the X value where the line will be drawn, and then select the Y parameter to determine which XZ plane it is on.

Draw line in YZ plane parallel to Y: Draws a line in the YZ plane. The user will select the Z value where the line will be drawn, and then select the X parameter to determine which YZ plane it is on.

Draw line in YZ plane parallel to Z: Draws a line in the YZ plane. The user will select the Y value where the line will be drawn, and then select the X parameter to determine which YZ plane it is on.

Draw Grid in XY plane: Draws a grid in the XY plane. The user will select the Z parameter to determine which XY plane it is on.

Draw Grid in XZ plane: Draws a grid in the XZ plane. The user will select the Y parameter to determine which XZ plane it is on.

Draw Grid in YZ plane: Draws a grid in the YZ plane. The user will select the X parameter to determine which YZ plane it is on.

4.11.5 Special Keys

Abort Plot: To STOP the plot at any time during the plotting or computation of the graph, press the $\boxed{\text{Esc}}$ key or $\boxed{\text{E}}$ key.

Pause Plot: To interrupt (PAUSE) the plot, press the $\boxed{\texttt{spacebar}}$. Pressing the spacebar again or any other key will RESUME the plot. The x-, y-, and z-coordinates of the last point plotted (in the current section being plotted) will be displayed.

4.11.6 Helpful Hints

1) Do not turn the hidden line option on (key $\boxed{\text{H}}$) until you finish your exploration because the plot will be slower with the hidden line option active.

2) Because of the increased plotting time, increase resolution only when you desire a "nice" plot.

3) To plot a *sphere*, for example, $z^2 + y^2 + x^2 = 70$ (and other similar quadratic surfaces), first plot $z = (70 - x^2 - y^2)^{(1/2)}$. Then select $\boxed{\text{F}}$ to change the "displayed" function to $z = -(70 - x^2 - y^2)^{(1/2)}$. Finally, click `previous menu` and the "lower" hemisphere will be overlayed on the existing "upper" hemisphere.

4) Use the zoom-in key $\boxed{\text{Z}}$ to change the cone of vision. Changing the distance d allows you to see more, or less, of the graph that is contained in the viewing box.

5) To zoom-in on features away from the center (aiming point) of a current graph, you may need to change both the viewing box *and* the aiming point. However, do this in low resolution until you are sure you have selected the viewing box and the aiming point that displays the desired features.